Review Article

Orbit Design Elements of Chang'e 5 Mission

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The three key orbit design technologies employed in the Chang'e 5 mission are identified and discussed in this paper: orbit design for lunar orbit rendezvous and docking, orbit design for precision lunar landing and inclination optimization, and orbit design for Moon-to-Earth transfer. First, an overview of the Chang'e 5 mission profile is presented, which is followed by detailed discussions of the three key orbit design technologies, including an introduction of the tracking-based orbit design methodology. Flight data are provided to demonstrate the correctness of the designs.

1. Introduction

In the early morning of November 24, 2020, the Chang'e 5 lunar probe was launched from the Wenchang Space Launch Center and successfully executed a 23-day journey of lunar sample return (LSR) mission. At the touchdown of the reentry capsule in the landing field of Inner Mongolia on December 17, 2020, the last doubt and many questions raised on the orbit of the Chang'e 5 mission during the design phase had been finally put to rest.

Prior to the Chang'e 5 mission, there had been nine LSR missions, including six Apollo missions of the United States and three Luna missions of the Soviet Union [1, 2]. However, since Luna 24 in 1976, there had been no LSR mission until the success of Chang'e 5. A major difference between the orbit designs of the Apollo and Luna missions is that the former included manned lunar orbit rendezvous and docking (RVD), whereas the Luna missions had no such flight phase. The apparent reason for this is that in the Apollo missions, it was simple too costly to carry the return module to the lunar surface and subsequently launch it to the lunar orbit in the absence of an RVD flight phase. A similar conclusion was reached in the propellant budget analysis for the Chang'e 5 mission during the early mission design phase. Therefore, based on the propellant budget analysis and a thorough review of the orbit designs of the Apollo missions, Luna missions, and Constellation Project [3], it was decided early in the design phase that the flight profile of Chang'e 5 would incorporate a lunar orbit RVD phase,

which makes Chang'e 5 the first space mission in the world that includes unmanned lunar orbit RVD.

Although there are quite a few successful LSR orbit designs for reference from the Apollo missions and Lunar missions, the orbit design for Chang'e 5 is by no means free of challenges. The most severe difficulty was related to the unmanned lunar orbit RVD, the first of its type in space history. In addition to the vast distance between the Earth and the Moon and the loss of tracking access when a spacecraft flies over the far side of the Moon, the number of tracking stations available for the orbit determination of the Chang'e 5 spacecraft was very limited. Thus, the RVD orbit scheme had to be carefully devised to ensure that the limited tracking resources were fully exploited, the control accuracy of the initial aim point of the RVD operation was guaranteed, and a smooth transition from the phasing stage to the far-range operation was achieved. A new orbit design methodology was needed to address these challenging problems. The constrained RVD fuel budget was another factor the orbit designers had to consider throughout the development of the RVD orbit scheme. Considering the multiple engineering constraints, the orbit scheme, such as maneuver sequence, maneuver locations, and lunar launch azimuth, had to be optimized to minimize the delta- V budget for the RVD operation.

Another important problem that had perplexed the orbit designers for a long time during the mission design was about the lunar launch: how can it be ensured that the ascent module is located in the orbit plane of the orbiter when it is launched from the lunar sample site two days after the lunar

landing, at the same time, the ascent module is closely monitored throughout the ascent process by the ground stations in China? The new orbit design methodology mentioned above played an important role here. In addition, it was found that the optimization of the lunar orbit inclination was the key algorithm for fully resolving this problem. This algorithm is coupled with the orbit design for precision lunar landing, which was yet another challenge posed by the mission goal of landing at the desired location in the area of Storm Ocean on the Moon.

Besides the lunar launch and the unmanned lunar orbit RVD operation, a major difference between the Chang'e 5 mission and previous Chang'e missions was that the orbiter and reentry capsule assembly had to leave the lunar orbit and enter the Moon-to-Earth transfer trajectory, a first for China's lunar exploration program (CLEP). Although the design techniques were documented well in the literature, the engineering design encountered several obstacles. It was found that the chosen 3000-N main engine enforcing trans-Earth insertion (TEI) maneuver may excite structural vibration, which could lead to a risky degradation of the control accuracy. In addition, late in the design phase it was found that the original orbit design for the Moon-to-Earth transfer may lead to an unacceptable reentry trajectory range.

This paper focuses on addressing these critical orbit design elements of the Chang'e 5 mission: orbit design for lunar orbit RVD, orbit design for precision lunar landing and inclination optimization, and orbit design for Moon-to-Earth transfer. First, an overview of the mission profile of Chang'e 5 is presented, followed by an introduction to the tracking-based orbit design methodology. Subsequently, detailed discussions of the above three key orbit design technologies are presented in the order of flight sequence. Flight data are provided to demonstrate the correctness of the designs.

2. Mission Profile

There are 11 flight phases in the Chang'e 5 mission: Earth launch, Earth-to-Moon transfer, lunar orbit insertion (LOI), lunar orbit flying with descent maneuver (DM), powered descent, lunar surface operation, phasing stage of RVD, far-range, close-range, and docking operations of RVD, lunar orbit waiting, Moon-to-Earth transfer, and reentry and recovery, as shown in Figure 1.

The Chang'e 5 spacecraft is composed of four modules: an orbiter, a lander, an ascent module, and a reentry capsule. After the spacecraft completes a 112 h Earth-to-Moon transfer, two LOI maneuvers are conducted with a one-day interval between the two LOIs, following which the spacecraft enters a near-circular lunar orbit with an altitude of approximately 200 km. After approximately 8 h, the lander and ascent module assembly is separated from the orbiter and reentry capsule assembly. Approximately one day after the second LOI, the lander and ascent module assembly performs two consecutive DMs to lower its perilune altitude to 15 km, and one day after the DMs, the assembly makes a powered descent to land at the desired location in the

Storm Ocean. After landing, the lunar sample collecting operation is conducted. Two days later, the ascent module is launched from the lunar surface, and within two days, four maneuvers are conducted by the ascent module before the initial aim point is achieved, when the ascent module is flying in a $210 \text{ km} \times 210 \text{ km}$ circular orbit, 50 km ahead of the orbiter along-track. The orbiter also performs four phasing maneuvers between the separation of the two assemblies and the lunar launch to ensure that it arrives at the desired orbital location at the prescribed time of the initial aim point. The far-range, close-range, and docking operations are completed in approximately 3.5 h after the initial aim point, followed by the transfer of the lunar sample from the ascent module to the reentry capsule. Subsequently, the ascent module is separated from the orbiter and reentry capsule assembly and eventually collides with the Moon surface, whereas the assembly continues to fly in a circular lunar orbit of altitude 210 km. After approximately 5–6 days, two TEI maneuvers are conducted with a one-day interval between them, and the orbiter and reentry capsule assembly enters the Moon-to-Earth transfer orbit. After 4–5 days, the reentry capsule is separated from the orbiter, reenters the atmosphere, and shortly lands in the chosen landing field of Inner Mongolia, whereas the orbiter is pulled up and heads to the Sun-Earth L2 point for an extended mission.

The launch of the Chang'e 5 mission was postponed several times owing to rocket problems. Eventually, it was decided that the mission would be launched in late 2020, and three launch windows were identified: November 24, 2020, November 25, 2020, and December 23, 2020 (Beijing time). The criteria (design constraints) used for the search of the launch windows included minimum delta- V budget, solar elevation of 30° – 50° during the lunar surface operation, and reentry flight range of 5600 km–7100 km. The actual launch day was November 24, 2020. The flight sequence for the launch window, November 24, 2020, is displayed in Figure 2.

Note that the LOI and the TEI are approximately 14 days (half a month in the lunar calendar) apart, which is because the lunar escape hyperbolic excess velocity vector returns to the initial lunar orbit plane every half a month and the delta- V needed for the orbit plane adjustment in the TEI can be minimized.

3. Lunar Orbit Design Based on Tracking Analysis

As mentioned earlier, the tracking resources for the Chang'e 5 mission were very limited, which posed a severe challenge in the orbit design. To fully exploit the tracking time potentially available for orbiter determination or maneuver monitoring, an orbit design methodology based on tracking analysis was developed in the early stage of the design phase of the mission [4–9].

Without loss of generality, referring to the top view from the north pole of the Earth, as shown in Figure 3, the ground tracking station is assumed to be located on the equator for simplicity of illustration. In addition, it is assumed that initially the lunar orbit of the spacecraft is invisible from the

FIGURE 1: Flight profile of Chang'e 5 mission.

ground station(s) on the Earth. At some instant, the tracking station moves to point A owing to the spin of the Earth, when the right side of the spacecraft orbit becomes visible to the station. Soon after, when the station is at point B, the left side of the orbit also becomes visible to the station. In Figure 3, the angular distance, θ , that the station travels when it moves from point A to point B is obtained as follows:

$$\theta = \frac{2r_{\text{orbit}}}{r_{\text{earth-moon}}} = 0.82^\circ, \quad (1)$$

where orbit radius $r_{\text{orbit}} = r_M + 1000 \text{ km} = 2737.4 \text{ km}$ is used in the calculation and $r_{\text{earth-moon}} = 384400 \text{ km}$ is the average distance between the Earth and the Moon.

The time required for the station to travel from point A to point B is found to be approximately 3.3 min (θ divided by the Earth spin rate), which is negligible compared to the duration when the Moon and the spacecraft orbit (not spacecraft) are continuously visible to the station. Therefore, the time instants at which the Moon and the spacecraft lunar orbit become visible to the station can be computed independently, regardless

FIGURE 2: Flight sequence of Chang'e 5 mission (launch window 2020-11-24, Beijing time).

FIGURE 3: Use of Moon center visibility time as natural time reference.

of the actual lunar orbit parameters, with only an error of a few minutes.

The same conclusion can be drawn for the time instants at which the Moon and the spacecraft orbit become invisible. As such, the time instants of the starting and ending of the tracking period of the ground station(s) can be computed

independently using the visibility of the Moon center, which constitutes a natural time reference in planning lunar orbit maneuvers in lunar missions.

Concurrently, if it is desired to monitor a maneuver process, the argument of latitude (AOL) of the maneuver also needs to be selected carefully, besides planning the

FIGURE 4: Initial orbit and landing site.

maneuver time. This can be achieved by selecting the AOL of the maneuver to be in the scope of the AOL of the lunar orbit that makes the spacecraft visible to the ground stations.

A detailed illustration of the lunar orbit design based on the tracking analysis is provided subsequently when the orbit design for the lunar orbit RVD is discussed.

4. Orbit Design for Precision Lunar Landing and Inclination Optimization

It had been demonstrated during the Apollo project era that precision landing on the lunar surface is achievable. In the Chang'e 5 mission design, several orbit design techniques were studied to realize this, considering the design constraints including the tracking access requirements.

There are many research papers in the literature that discuss orbit designs for precision landing on the lunar surface. By reviewing past experiences, it is found that there are two key elements in the orbit design for precision landing on the Moon, namely, a two-to-two targeting maneuver for achieving the desired perilune altitude and landing latitude, and an orbit plane adjustment or a phasing maneuver for achieving the desired longitude of the landing site. Concurrently, it is essential to use a high-fidelity descent dynamic model and have accurate knowledge of the guidance law in the design process to accurately compute the nominal descent range [10, 11].

The following discussion is adapted from our previous studies on the precision lunar landing and inclination optimization for the Chang'e 5 mission [6, 7], which is followed by a brief elucidation of the actual flight data.

4.1. Problem Statement. Referring to the Moon-centered celestial sphere, as shown in Figure 4, following the LOI

maneuvers, initially the lander and the orbiter are on a 200 km-altitude near-circular lunar orbit, with an orbit inclination of approximately 43.7° . Moreover, the desired latitude and longitude of the landing site are specified.

The task is to achieve an elliptical orbit starting from the initial 200 km-altitude circular orbit to deliver the lander and ascent module assembly to the descent start point at the perilune of the elliptical orbit. This should achieve the desired altitude (typically 15 km) and AOL to ensure a successful powered descent onto the lunar surface near the specified landing site. Note that the nominal perilune altitude at the decent start point is chosen to be approximately 15 km for the safety of the lander because the height of highest mountain on the Moon is about 10 km. For the actual flight, the perilune altitude at the decent start point is adjusted to 15.568 km with respect to the Moon radius 1737.4 km (18 km with respect to the landing site), based on the predicted perilune altitude variation after the DMs.

Referring to point A in Figure 4, the intended landing site is close to the orbit plane when the lander begins the powered descent at descent point E, and the landing site will be in the descent trajectory plane when the lander lands, owing to the spin of the Moon.

Referring to Figure 5, in the following discussion, it is assumed that initially the lander is flying in a $200 \text{ km} \times 200 \text{ km}$ circular orbit. The spacecraft remains in this orbit for several revolutions for system checkout and orbit determination. Subsequently, at an appropriate AOL, a transverse burn Δv_T is conducted to place the lander in an elliptical orbit with a perilune altitude of 15 km. After several revolutions of the orbit for system checkout and orbit determination, when the lander reaches the descent point with the desired altitude (15 km) and AOL, it starts to follow a guided descent trajectory onto the lunar surface.

FIGURE 5: Basic orbit geometry.

Note that accurate knowledge of the nominal descent range and descent time is available from the design iteration with the design team of the powered-descent guidance.

4.2. Orbit Inclination Optimization. The optimization of the lunar orbit inclination was the key algorithm in the nominal orbit design for the Chang'e 5 mission. Referring to Figure 4, consider a lunar surface stay time of two days and in-plane ascent (point B). Applying spherical geometry, it can be shown that the approximate value of the lunar orbit inclination is obtained by the following equation:

$$\tan i = \frac{\tan \varphi}{\cos(\omega_m t_{AB}/2)}, \quad (2)$$

where $\varphi = 43.11^\circ$ is the specified latitude of the landing site in this study, ω_m is the spin rate of the Moon, and t_{AB} is the time that the landing site travels from point A to point B in Figure 4 (the lunar surface stay time of two days). Using the above formula, the orbit inclination is found to be $i = 43.7^\circ$.

The approximate value just found is used as the initial value of the lunar orbit inclination in the nominal orbit design. The optimization of the lunar orbit inclination plays a central role in the overall nominal orbit design for the sample return mission. When the sample site passes the orbit plane for the second time at point B, the ascent module will take off from the lunar surface in the orbit plane of the orbiter. The objective of the orbit inclination optimization is to achieve a coplanar ascent, i.e., to ensure the desired sample site passes through the orbit plane of the orbiter (point B) at the desired ascent time. In the Chang'e 5 mission, the desired ascent time was chosen as the start time when the Moon center (and the powered ascent) becomes visible from the primary tracking stations plus 10 min. A coplanar launch is chosen to minimize the fuel consumption for the orbit plane correction. Differential correction can be employed to achieve optimal orbit inclination. An illustration of the procedure of the nominal orbit design for the

sample return mission with orbit inclination optimization is provided subsequently (Figure 6).

4.3. Orbit Control Strategy for Precision Lunar Landing. For a successful descent and precision landing at the desired location on the lunar surface, appropriate values of the altitude and AOI at the descent point have to be achieved, and when the lander touch downs on the lunar surface, the intended landing site reaches the descent trajectory plane with the spin of the Moon.

In the Chang'e 5 mission design, two key elements were identified in the orbit control strategy for precision lunar landing. One element is a two-to-two maneuver for targeting the descent point conditions to ensure the correct latitude of the landing site and descent point altitude are achieved, and the other element is an orbit plane adjustment maneuver for achieving the desired longitude of the landing site.

4.3.1. Two-to-Two Targeting Maneuver. To achieve precision landing at the desired landing site, the lander needs to be at the correct AOL and altitude when it reaches the descent point. This can be achieved using a two-to-two targeting maneuver, and the descent altitude is typically specified to be 15 km from the lunar surface at periselene [11].

Referring to Figure 4, the desired AOL of the descent point can be computed as follows. Because both the latitude of the landing site and inclination of the orbit are known (from navigation update), the spherical angle, $\widehat{AD}$, can be computed from the sine law as follows:

$$\frac{\sin \widehat{AD}}{\sin 90^\circ} = \frac{\sin \varphi}{\sin i}. \quad (3)$$

As discussed earlier, for a lunar surface stay of two days, the orbit inclination is $i = 43.7^\circ$ for the example considered in this paper, and the above equation yields $\widehat{AD}$. Assuming that the nominal descent range is accurately known (e.g., 600 km) from the descent trajectory design, the AOL of the descent point is obtained from the following equation:

$$u_E = \widehat{AD} - \frac{600 \text{ km}}{r_M} \cdot \frac{180^\circ}{\pi}, \quad (4)$$

where r_M is the radius of the Moon. From Equation (3) and Equation (4), the AOL of the descent point is found to be $u_E = 61.8^\circ$. Note that the descent cross range is assumed to be zero; therefore, the descent trajectory plane is assumed to be the same orbit plane at the decent point (point E in Figure 4).

To achieve the desired descent start point condition, i.e., descent point altitude $h_E = 15\text{km}$ and desired AOL u_E (or desired latitude of the landing site φ), two design parameters are used: magnitude of the transverse burn Δv_T and AOL u_T at which maneuver Δv_T occurs. In other words, maneuver Δv_T in Figure 5 is a two-to-two targeting maneuver: $(\Delta v_T, u_T) \rightarrow (h_E = 15\text{km}, \varphi = 43.11^\circ)$.

FIGURE 6: Core algorithm of precision lunar landing and inclination optimization.

Referring to Figure 5, the approximate values of Δv_T and u_T can be determined as follows:

$$u_T = u_E + 180^\circ, \quad (5)$$

$$\Delta v_T = \sqrt{\frac{\mu_m}{r_T}} - \sqrt{2 \left(\frac{\mu_m}{r_T} - \frac{\mu_m}{2a} \right)}, \quad (6)$$

where r_T is the orbit radius of the 200 km-altitude circular orbit and a is the semimajor axis of the 15 km $\times$ 200 km elliptical orbit. From Equation (5) and Equation (6), the approximate values of Δv_T and u_T are found as follows: $u_T = 241.8^\circ$, $\Delta v_T = 40.4$ m/s.

Once the approximate values of the two design parameters are found, they are used as the initial values to start the iteration process of the differential correction. The precise values of the two design parameters are found using the two-to-two targeting sequence: $(\Delta v_T, u_T) \rightarrow (h_E = 15$ km, $\varphi = 43.11^\circ)$. The detailed computation algorithm is given in the following. The algorithm was adapted from the paper presented by the author in the 2011 AAS Space Flight Mechanics Meeting. Moreover, the readers can refer to Reference [12] for an introduction to the basic algorithms of differential correction.

The $(\Delta v_T, u_T)$ values obtained above are approximate, but they are good starting values for computing the desired precise values using an iteration procedure. The iteration procedure is based on the small deviation assumption; namely, the approximate starting values of $(\Delta v_T, u_T)$ are assumed to be close to the precise values.

The design parameters and targeted variables are denoted as follows:

$$\begin{aligned} \bar{p} &= \begin{pmatrix} \Delta v_T \\ u_T \end{pmatrix}, \\ \bar{q} &= \begin{pmatrix} h_E \\ \varphi \end{pmatrix}, \end{aligned} \quad (7)$$

where $\bar{p}$ is the design parameters at the descent maneuver and $\bar{q}$ is the corresponding targeted variables that is obtained via numerical integrations of a high-fidelity orbital dynamics model (orbit propagation). The desired value of $\bar{q}$ is $(h_E = 15$ km, $\varphi = 43.11^\circ)$ and denoted as $\bar{q}_d$. The targeted variables $\bar{q}$ can be treated as a function of the design parameters $\bar{p}$. Based on the small deviation assumption, it can be written

as follows:

$$\bar{q} = \bar{q}(\bar{p}) \geq \Delta\bar{q} = \frac{\partial\bar{q}}{\partial\bar{p}}\Delta\bar{p}, \quad (8)$$

where $\Delta\bar{q} = \bar{q} - \bar{q}_d$, $\Delta\bar{p} = \bar{p} - \bar{p}_d$, and $\bar{p}_d$ are the desired values of $\bar{p}$, namely, the precise values of the two design parameters to be found.

The iteration starts with the orbital propagation from the decent maneuver to landing on the Moon, using the approximate values of $(\Delta v_T, u_T)$ found earlier. When the integration finishes, $\Delta\bar{q} = \bar{q} - \bar{q}_d$ is computed. If the error transfer matrix $\partial\bar{q}/\partial\bar{p}$ is available, then the correction for the design parameters is as follows:

$$\Delta\bar{p} = \left[\frac{\partial\bar{q}}{\partial\bar{p}} \right]^{-1} \Delta\bar{q}. \quad (9)$$

Therefore the new value of the design parameters at the descent maneuver is $\bar{p}_{\text{new}} = \bar{p} - \Delta\bar{p}$.

If the new design parameters still lead to a significant end error $\Delta\bar{q}$, then a new error transfer matrix is computed and a new correction $\Delta\bar{p}$ is found. This process is repeated until the end error becomes negligible, when the iteration converges and the precise values of the two design parameters are given by $\bar{p}_d = \bar{p}_{\text{new}} = \bar{p} - \Delta\bar{p}$.

In the following, a numerical method is introduced to compute the error transfer matrix. Denote the approximate value of the design parameters as $\bar{p}_0 = (\Delta v_{T0}, u_{T0})^T$ and the value of the targeted variables corresponding to $\bar{p}_0$ as $\bar{q}_0$. Introducing a small deviation to each of the two components of $\bar{p}_0$ gives the following equation:

$$\begin{aligned} \bar{p}_0 &= \begin{pmatrix} \Delta v_{T0} \\ u_{T0} \end{pmatrix}, \\ \bar{p}_1 &= \begin{pmatrix} \Delta v_{T0} + \delta(\Delta v_{T0}) \\ u_{T0} \end{pmatrix}, \\ \bar{p}_2 &= \begin{pmatrix} \Delta v_{T0} \\ u_{T0} + \delta(u_{T0}) \end{pmatrix}. \end{aligned} \quad (10)$$

Note that the typical values of the small deviations can be taken as $\delta(\Delta v_{T0}) = 0.001 \text{ m/s}$, $\delta(u_{T0}) = 0.001^\circ$. Then the corresponding targeted variables $\bar{q}_0$, $\bar{q}_1$, and $\bar{q}_2$ are computed via orbital propagation, respectively.

Since $\Delta\bar{q} = (\partial\bar{q}/\partial\bar{p})\Delta\bar{p}$, it can be written as follows:

$$(\bar{q}_0 - \bar{q}_d) = \frac{\partial\bar{q}}{\partial\bar{p}}(\bar{p}_0 - \bar{p}_d), \quad (11)$$

$$(\bar{q}_1 - \bar{q}_d) = \frac{\partial\bar{q}}{\partial\bar{p}}(\bar{p}_1 - \bar{p}_d), \quad (12)$$

$$(\bar{q}_2 - \bar{q}_d) = \frac{\partial\bar{q}}{\partial\bar{p}}(\bar{p}_2 - \bar{p}_d). \quad (13)$$

The subtractions of Equation (12)–Equation (11) and Equation (13)–Equation (11) give the following:

$$\begin{aligned} (\bar{q}_1 - \bar{q}_0) &= \frac{\partial\bar{q}}{\partial\bar{p}}(\bar{p}_1 - \bar{p}_0), \\ (\bar{q}_2 - \bar{q}_0) &= \frac{\partial\bar{q}}{\partial\bar{p}}(\bar{p}_2 - \bar{p}_0). \end{aligned} \quad (14)$$

The above two equations can be written in matrix form as follows:

$$\begin{bmatrix} \bar{q}_1 - \bar{q}_0 & \bar{q}_2 - \bar{q}_0 \end{bmatrix} = \frac{\partial\bar{q}}{\partial\bar{p}} \begin{bmatrix} \bar{p}_1 - \bar{p}_0 & \bar{p}_2 - \bar{p}_0 \end{bmatrix}. \quad (15)$$

Denoting $P = [\bar{p}_1 - \bar{p}_0 \quad \bar{p}_2 - \bar{p}_0]$ and $Q = [\bar{q}_1 - \bar{q}_0 \quad \bar{q}_2 - \bar{q}_0]$, the above equation is rewritten as follows:

$$Q = \frac{\partial\bar{q}}{\partial\bar{p}}P \implies \frac{\partial\bar{q}}{\partial\bar{p}} = QP^{-1}. \quad (16)$$

Since $P = [\bar{p}_1 - \bar{p}_0 \quad \bar{p}_2 - \bar{p}_0] = \begin{bmatrix} \delta(\Delta v_{T0}) & \\ & \delta(u_{T0}) \end{bmatrix}$, the

inverse P^{-1} can be readily computed.

Therefore, instead of deriving the error transfer matrix analytically, this numerical method requires only numerical integrations of the orbital dynamics (orbital propagation) to find the error transfer matrix. Moreover, it can be seen from the preceding discussion that this numerical approach for differential correction can be easily extended to the scenarios for which more design parameters and targeted variables are present, e.g., the four-to-four targeting sequences needed in the subsequent discussions.

4.3.2. Orbit Plane Adjustment. In the above discussions, it was assumed that the desired landing site is in the descent trajectory plane when the lander lands. The above two-to-two maneuver targets the latitude of the intended landing site (φ). For precision lunar landing, the longitude of the landing site also needs to be targeted in the orbit control.

The phasing orbit is considered to help achieve the desired landing longitude in the Chang'e 5 mission design. It is found that because the time available for phase angle adjustment is limited, the phasing strategy is impractical [6, 7].

Instead, to achieve the desired longitude of the landing site, the strategy of adjusting the orbit plane is adopted in the mission design. Referring to Figure 4, the objective of the orbit plane change is to shift point A to the right (or left) along the same latitude line such that when the lander is at point T to make maneuver Δv_T , the landing site is $t_{TE} + 15 \text{ min}$ away from the intersection with the orbit plane. The shift of point A along the latitude line can be described by the change in its right ascension α_A .

There are a few alternatives for achieving such a plane change. The first option is to adjust the RAAN. Referring to Figure 4, if only the RAAN (Ω) of the $200 \text{ km} \times 200 \text{ km}$ circular orbit is adjusted (at $u = 90^\circ$ or 270°), points D and A will rotate about the spin axis of the Moon for the same

FIGURE 7: Inclination adjustment.

angle, i.e.,

$$\Delta\Omega = \Delta\alpha_A. \quad (17)$$

From the following VOP equation:

$$\Delta\Omega = \frac{r \sin u}{na^2 \sqrt{1-e^2} \sin i} \cdot \Delta v_n, \quad (18)$$

the velocity impulse, Δv_n , in the orbit normal direction can be obtained from the above equation.

The second option is to adjust the inclination. The desired shifting of point A along the latitude line can be also achieved by adjusting the inclination; referring to Figure 7, a change in the inclination shifts the landing site from point A to A'.

Referring to the spherical triangle, ACD, in Figure 7, from the sine and cosine laws,

$$\sin \widehat{AD} = \frac{\sin \varphi}{\sin i}, \quad (19)$$

$$\cos \widehat{AD} = \cos \varphi \cos \widehat{DC} = \cos \varphi \cos (\alpha_A - \Omega). \quad (20)$$

Since $\sin^2 \widehat{AD} + \cos^2 \widehat{AD} = 1$, the above two equations can be combined as follows:

$$\left(\frac{\sin \varphi}{\sin i} \right)^2 + \cos^2 \varphi \cos^2 (\alpha_A - \Omega) = 1. \quad (21)$$

Differentiation yields the following

$$\Delta i = - \frac{\sin^3 i}{2 \sin^2 \varphi \cos i} \cos^2 \varphi \sin 2(\alpha_A - \Omega) \cdot \Delta \alpha_A. \quad (22)$$

Note that $\widehat{AD}$ can be computed from Equation (19), and

subsequently, $\widehat{DC} = \alpha_A - \Omega$ can be obtained from Equation (20). Applying the following VOP equation ($u = 0^\circ$ or 180°):

$$\Delta i = \frac{r \cos u}{na^2 \sqrt{1-e^2}} \cdot \Delta v_n, \quad (23)$$

the velocity impulse, Δv_n , applied in the orbit normal direction can then be found from the above equation.

The third option is to adjust both the inclination and RAAN. In this case, the changes in the right ascension of point A due to the RAAN and the inclination adjustments are given by Equations (17) and (22), respectively, and the total change in the right ascension of point A is found to be

$$\Delta \alpha_A = \Delta \Omega - \frac{2 \tan^2 \varphi \cos i}{\sin^3 i \sin 2(\alpha_A - \Omega)} \cdot \Delta i. \quad (24)$$

Substituting Equations (18) and (23) into the above equation yields the following:

$$\Delta \alpha_A = \frac{r}{na^2 \sqrt{1-e^2} \sin i} \cdot \left[\sin u - \frac{2 \tan^2 \varphi \cos i}{\sin^2 i \sin 2(\alpha_A - \Omega)} \cos u \right] \cdot \Delta v_n. \quad (25)$$

This equation can be used to compute Δv_n when the plane change occurs at an arbitrary AOL, u . Therefore, the plane change can be combined with maneuver Δv_T at point T. The Δv magnitude of the combined maneuver is given by $\sqrt{\Delta v_T^2 + \Delta v_n^2}$. When Δv_T is sufficiently large, there is no significant increase in the fuel consumption for the plane change.

Because in the third option, there are no additional maneuver and no significant increase in delta-V for the plane change, the method of adjusting both the inclination and RAAN is adopted. Specifically, it is proposed that a normal component of the velocity impulse is added in the LOI to realize the adjustment of the orbit plane and target the desired landing longitude, referring to the following discussion.

4.4. Orbit Scheme for Precision Lunar Landing and Inclination Optimization. As mentioned above, the core algorithm of the orbit design for the Chang'e 5 mission involved the optimization the lunar orbit inclination, namely, the lunar orbit inclination, which is targeted during the Earth-to-Moon transfer, should be optimized such that the prescribed landing site passes through the orbit plane of the orbiter at the prescribed ascent time, which is selected to ensure that the lunar ascent has good tracking access condition and the lunar surface operation time is approximately 2 days. On the other hand, the adjustment of the lunar orbit inclination is coupled with the realization of precision lunar landing; therefore, the orbit scheme for precision lunar landing has to be devised in combination with the optimization of the lunar orbit inclination. Refer to Figure 6 for an illustration of the detailed algorithm.

As can be seen from the figure, the targeting of the desired landing longitude is combined with the LOI

(adjusting the normal component of the LOI velocity impulse), and there is no additional maneuver for the plane change to target the landing longitude. Moreover, the targeted lunar orbit inclination is adjusted during the Earth-to-Moon transfer to achieve a coplanar ascent at the prescribed time. These two differential correction processes are combined into a single outer loop two-to-two (denoted as 2×2) differential correction process for rapid iteration convergence. The targeting of the desired landing latitude and altitude of the descent start point constitutes the inner loop 2×2 differential correction.

A major challenge and design constraint for planning precision soft-landing on the lunar surface is the limited number of ground tracking stations involved in the operation. The timing of the maneuvers needs to be carefully planned, and the tracking analysis based lunar orbit design method is applied, referring to the preceding discussions. In the Chang'e 5 mission, the orbiter and the lander were controlled from the ground stations separately, and the lander is visible to the Earth during the powered descent and entire lunar surface operation since the landing site is on the near side of the Moon. To ensure the powered descent and the lunar surface operation right after landing have sufficient support from Chinese ground stations, the start of the powered descent is scheduled to be the first perilune passage after the start of the tracking period of the Moon center for the primary ground stations in China. In contrast, for a lunar sample return mission with the landing site on the far side of the Moon, the use of a single probe located over an assistance orbit around the Moon is recommended, in which the assistance orbit with quasi-constant values of semimajor axis, eccentricity, and inclination can be designed for improved lunar landing performance in terms of mission costs, trajectory determination, and visibility with respect to a probe positioned in the Lagrangian point L2 of the Earth-Moon system [13].

4.5. Flight Test Results. Table 1 lists the longitude and latitude of the nominal landing site and the actual landing site location in the actual flight of the Chang'e 5 mission. Apparently, the landing site longitude error is approximately 0.5° , and the landing site latitude error is about 0.05° , which means that the actual landing site was approximately 11 km away (to the west) from the nominally chosen landing site.

However, the orbit design for the precision lunar landing and the orbit inclination optimization is still considered to be highly successful for the following reasons:

First, the actual landing site was sufficiently close to the nominal landing site; therefore, no significant orbit plane correction ΔV was needed in the lunar orbit RVD operation. Based on the dispersion analysis in the mission design, a maximum of 25 km landing position error in the longitude direction was estimated in the absence of accurate calibration of the descent main engine thrust.

Second, during the actual flight test, the calibration of the 7500-N main engine thrust yielded the lower and upper bounds of the thrust magnitude. The higher thrust magni-

TABLE 1: Landing site longitude and latitude.

	Longitude ($^\circ$)	Latitude ($^\circ$)
Nominal design	-51.42	43.11
Flight data	-51.9161	43.0617
Difference	-0.4961	-0.0483

tude was purposely adopted in the guidance update because a shorter descent range was desired as there is a lunar creek on the lunar surface near the predicted landing site, and this creek was located in the flight direction beyond the predicted landing site, which was a landing safety hazard and had to be avoided.

4.6. Summary. The orbit inclination optimization is identified as the core algorithm of the nominal orbit design for the sample return mission, and the two key elements in the orbit design for precision landing on the Moon are discussed in detail. It is shown that a two-to-two targeting maneuver can be utilized to achieve the desired descent point altitude and the desired landing point latitude. It is also exhibited that orbit plane adjustment combined with LOI can be used for achieving the desired longitude of the landing site.

It is demonstrated that the orbit scheme for precision lunar landing had to be devised in combination with the optimization of the lunar orbit inclination in Chang'e 5 mission. Actual flight data are provided to verify the correctness and effectiveness of the orbit design for precision lunar landing and inclination optimization.

5. Orbit Design for Lunar Orbit Rendezvous and Docking

The orbit design for lunar orbit RVD includes the phasing orbit designs of both the ascent module and the orbiter. In Figure 8(a), the great circle passing through points A and B is the projection of the lunar orbit on the Moon-centered celestial sphere. With the rotation of the Moon, the selected sample site, S, passes through the orbit plane at point A, when the lander and ascent module assembly conduct the powered descent and land at the sample site. After two days, the lunar sample collecting operation is completed, and the sample site passes through the orbit plane for the second time, when the ascent module lifts off from the lunar surface and continues to perform several phasing maneuvers to achieve the lunar orbit rendezvous with the orbiter.

The phasing stage of the ascent module starts at the lunar orbit insertion, when the ascent module enters the $15 \text{ km} \times 180 \text{ km}$ initial lunar orbit. Subsequently, the ascent module performs several orbit maneuvers that are controlled from the ground stations, and the phasing stage ends when the prescribed initial aim point is achieved. On reaching the aim point, on-board relative navigation capability should have been achieved, and the far-range operation starts. This is followed by close-range and docking operations. The design of the maneuver strategy in the phasing stage was

FIGURE 8: Concept of lunar orbit rendezvous.

the focus of the orbit design for the lunar orbit RVD in the Chang'e 5 mission.

5.1. Problem Statement. The objective of the orbit design for the RVD phasing stage of the Chang'e 5 mission was for the prescribed AOL to be achieved by the ascent module in the circular orbit of altitude 210 km at the prescribed time of the initial aim point; in addition, the orbiter was required to execute several maneuvers before the lunar ascent of the ascent module to ensure that it arrives at the prescribed AOL in the circular orbit of altitude 200 km at the same prescribed time of the initial aim point; moreover, the ascent module must be 50 km ahead of the orbiter along-track at the time of the initial aim point (Figure 8(b)). At the initial aim point, the control accuracy of the relative position and velocity between the orbiter and the ascent module must meet the criteria for the transition from the phasing stage to the far-range operation. Note that it was required that the ascent module completes the phasing in 1–2 days following the lunar ascent because the tracking condition for the far-range, close-range, and docking operations would become undesirable if it takes too long to reach the initial aim point.

As mentioned in Introduction, the difficulties in the lunar orbit RVD of the Chang'e 5 mission were (1) to fully exploit the limited tracking resources to achieve the required control accuracy of the initial aim point and (2) to minimize the total delta- V to meet the propellant budget constraint. Specifically, tracking access should be available for the lunar ascent, orbital maneuvers, initial aim point, and docking point; especially, it is required that the lunar ascent, initial aim point, and docking point should have tracking access support from the ground stations in China. Moreover, there should be sufficient tracking arcs for orbit determination between two adjacent maneuvers to ensure certain orbit

determination accuracy. The orbit control accuracy of the initial aim point should meet the requirement for the transition to the far-range operation (autonomous control without ground support). Furthermore, the time between the initial aim point and the docking point should be 3.5 h, and the duration of the tracking arc containing the docking point should be no less than 76 min.

To overcome these difficulties and satisfy the design constraints, the earlier described lunar orbit design method based on tracking analysis is employed. Furthermore, the orbit scheme is optimized in terms of the number of maneuvers, maneuver sequence, and maneuver position to achieve the minimum total delta- V , considering the tracking constraints. The following discussion is based on or adapted from References [4, 5, 14].

5.2. Phasing Orbit Scheme Selection. The main objectives of the phasing stage of a rendezvous mission are to reduce the phase angle difference between the chaser (ascent module) and the target spacecraft (orbiter) as well as adjust the orbit plane, altitude, and shape of the chaser orbit. Four different phasing orbit schemes were considered in the Chang'e 5 mission design [4, 5].

The two-impulse phasing orbit scheme is a rapid rendezvous scheme similar to that adopted by the Apollo missions or the Constellation Project [15, 16], in which the ascent module performs an orbit maneuver (Δv_1) at the first apselene passage after entering the initial $15 \text{ km} \times 180 \text{ km}$ lunar orbit. After half a revolution, the ascent module performs a comprehensive correction maneuver (Δv_2) and then coasts to the initial aim point. For this type of phasing strategy, there is actually no maneuver for the phasing. Instead, there is a rigorous requirement for the initial phase angle difference between the two spacecrafts when the chaser enters the lunar orbit at the perilune; namely, the width of the lunar ascent

FIGURE 9: Four-impulse scheme.

window is zero to achieve the optimal fuel consumption. It is shown that an additional delta- V of 10 m/s is needed for an extended lunar ascent window of approximately 1 min [16]. Moreover, in the two-impulse scheme, each impulse has two components: one transverse component for in-plane orbital parameter correction and one normal component for the orbit plane correction; the AOLs of the two maneuvers are close to 90° or 270° , which are very inefficient for the correction of inclination. Concurrently, on-board relative navigation is essential for rapid lunar orbit rendezvous missions. The maximum range of the Apollo rendezvous radar is approximately 400 nm; therefore, relative navigation was introduced early in the rendezvous operation. If the maximum range of the relative navigation device cannot meet the needs in the phasing stage, then the determination of the position and velocity of the vehicle can only rely on an inertial navigation system (IMU), which can yield unacceptably large position and velocity errors at the aim point. Therefore, a two-impulse scheme was not adopted for the Chang'e 5 mission.

In the three-impulse scheme, there is an additional impulse for phasing. However, similar to the two-impulse scheme, there is no independent impulse for the orbit plane correction. Instead, the two impulses for the in-plane orbital parameter correction both have a normal component for the orbit plane correction, which is very inefficient for the correction of inclination. Therefore, the three-impulse scheme was not adopted for the Chang'e 5 mission.

In contrast, it is found that the four-impulse scheme has a better lunar ascent window (because of the inclusion of a

phasing maneuver) and better fuel efficiency for the orbit plane correction (because of the inclusion of an independent maneuver for the orbit plane correction). Referring to Figure 9, the ascent module achieves a $15 \text{ km} \times 180 \text{ km}$ elliptical orbit after the lunar ascent and orbit insertion. Subsequently, a transverse burn (Δv_1) at the apolune will put the ascent module on a $180 \text{ km} \times X \text{ km}$ phasing orbit to adjust the phase angle difference between the ascent module and the orbiter. Following this, a plane correction maneuver (Δv_2) is conducted, which is perpendicular to the orbit plane. The last two transverse burns (Δv_3 and Δv_4) adjust the size and shape of the orbit. Eventually, the ascent module reaches a $210 \text{ km} \times 210 \text{ km}$ target orbit and achieves the required AOL at the prescribed time of the initial aim point.

The four-impulse scheme includes a maneuver dedicated to adjusting the orbital plane, referring to the normal velocity impulse, Δv_2 , in Figure 9. The other three maneuvers ($\Delta v_1, \Delta v_3$, and Δv_4) are all in transverse directions and are called the phasing maneuver, altitude maneuver, and circularization maneuver, respectively. Compared with the normal impulse components for the orbital plane adjustment in the three-impulse scheme, the independent velocity impulse, Δv_2 , is energy optimal in adjusting the orbital plane, and its AOL can be computed as follows:

$$\tan u = \tan(u + \pi) = \frac{\Delta\Omega \sin i}{\Delta i}. \quad (26)$$

There are two solutions for the AOL, which are 180° apart. Although the specific value of the AOL of Δv_2

FIGURE 10: Tracking and lighting conditions of four-impulse scheme.

depends on the corrections desired for the RAAN and inclination ($\Delta\Omega$ and Δi), at least one of the two AOL solutions in the above equation can be selected for applying Δv_2 , and the maneuver can always be monitored from the ground stations.

If the velocity impulse for the orbit plane adjustment in the four-impulse scheme is replaced by two velocity impulses, both applied in the normal direction—one for correcting the inclination and the other for correcting the RAAN—then a five-impulse scheme is obtained. The other three impulses are the same as in the four-impulse scheme. However, a close examination shows that for the five-impulse scheme, the ground tracking condition for the orbit determination before the third maneuver is poor and the orbit determination accuracy is unsatisfactory, which lead to a significant position error in the orbit normal direction at the aim point. Therefore, the five-impulse scheme was not adopted for the Chang'e 5 mission.

In summary, the two-, three-, and five-impulse schemes were found to be unsuitable for the Chang'e 5 mission, and thus, the four-impulse scheme was selected as the baseline scheme in the nominal orbit design for the lunar orbit RVD for the Chang'e 5 mission.

5.3. Optimization of Nominal Orbit Design. The four-impulse baseline design is essentially a special point maneuver scheme. The trajectory is optimal in terms of the fuel consumption; however, the tracking access may be unavailable for some maneuvers, and the transverse burn, Δv_4 , may be too large in magnitude, which can cause significant orbit parameter dispersions at the aim point, etc. An optimal nominal orbit design can be obtained after these practical engineering constraints have been taken into consideration.

The tracking condition analysis for the phasing orbit design of the ascent module is presented in Figure 10. In the figure, Station I refers to the primary China tracking stations and Station II refers to the tracking stations located in a foreign country. The bars over the horizontal axis show the

time spans in which the Moon center is visible to the stations. Time point "0" refers to the instant when the Moon center becomes visible from Station I. The two groups of solid lines represent the AOL boundary values, providing the AOL scope in which the tracking access is available. The scope of the AOL values at which the spacecraft is in the shadow is also shown in Figure 10, referring to the shaded region.

In the orbit design, each maneuver should be executed in the time spans in which the Moon center is visible to the stations, and the locations of the maneuvers should be in the AOL scope in which the tracking access is available, i.e., the AOLs of the maneuvers should be between the two AOL boundary values.

To ensure the precision of orbit determination before each maneuver, typically, two revolutions are needed for orbit measurement using the ground tracking stations, one additional revolution is needed for uploading the maneuver command, and the actual maneuver is executed in the fourth revolution. Therefore, there are at least four revolutions between two adjacent maneuvers. Considering these design requirements and using Figure 10 as a reference in the arrangement of the maneuvers, the final arrangement of the number of revolutions for each maneuver in the phasing orbit design is shown in Figure 11. In the figure, the number below the horizontal axis is the number of orbit revolutions. Note that the orbit period of the lunar orbit is approximately 2 h. This arrangement considers the tracking access requirements for the orbit insertion and the aim point. As can be seen from the figure, two revolutions of the orbit are reserved for the orbit determination after the final maneuver, which provides the inertial guidance input for the far-range rendezvous stage.

Monitoring the maneuvers by the ground stations is desired; therefore, the AOLs for which the tracking access is available should be chosen for the maneuvers. Moreover, it is better for some maneuvers to be performed in the shadow to allow the spacecraft to have more time to charge

FIGURE 11: Arrangement of number of revolution.

its battery when it is illuminated after coming out of the shadow. Thus, the optimal AOLs of the maneuvers in the four-impulse scheme can be determined by searching within the eligible scope of the AOL, as shown in Figure 10. This ensures the desired tracking conditions for the four orbit maneuvers and that some of them occur in the shadow. In addition to the minimum total delta- V requirement, the practical engineering constraints are considered, and the main optimization constraints are summarized as follows:

- (1) The AOL for each maneuver should be close to the AOLs of the special points and chosen between the upper and lower boundary values, as shown in Figure 10
- (2) The time of each maneuver should be chosen in the time span when the orbit is visible from the ground stations (Stations I or II), as indicated in Figure 10
- (3) Each transverse delta- V should be greater than 1 m/s and should be in the direction of flying to minimize the total delta- V and avoid a large attitude maneuver
- (4) The first maneuver, Δv_1 , should be greater than 30 m/s to achieve a higher orbit for a better battery charge condition, and the last maneuver, Δv_4 , should be less than 10 m/s to avoid a large control error and resulting large aim point dispersions

In the optimization process, the search scope of the AOL for each maneuver is chosen first, according to the above optimization constraint. For example, since the AOL of the perilune at the insertion is 108° , the AOL of the apolune of the initial orbit is $108^\circ + 180^\circ = 288^\circ$; therefore, the search scope of the AOL for the first maneuver Δv_1 can be chosen between $288^\circ - 50^\circ = 238^\circ$ and $288^\circ + 50^\circ = 338^\circ$; however, considering the AOL boundary values in Figure 10, the search scope of the AOL for the first maneuver is determined to be $246^\circ - 281^\circ$. In a similar way, the search scopes of the AOLs for the other three maneuvers (Δv_2 , Δv_3 , and Δv_4) were chosen as $50^\circ - 285^\circ$, $56^\circ - 158^\circ$, and $249^\circ - 296^\circ$, respectively.

Subsequently, a grid search procedure is employed to find the optimal solutions of the AOLs and corresponding delta- V s. Specifically, for each combination of the AOL values from the search scopes, the precise value of each maneuver is computed as follows: the first step is to obtain the approximate values of the four maneuvers, for which Baranov's theory or the methods based on the VOP equa-

tions can be utilized [17–19]; once the approximate values are obtained, they are substituted into a two-body model for an iteration process (differential correction); when the iteration converges, better approximate values are achieved, and they are substituted into the accurate perturbation model for further iteration, which yields the precise value of each maneuver. For a flow chart and further discussion on this method of computing the precise values of the maneuvers for each combination of the AOL values, refer to Reference [4]. When the grid search is completed, only the solutions that satisfy all the preceding optimization constraints have been retained and are treated as the valid candidates of the nominal orbit.

Taking the first launch window of the Chang'e 5 mission (November 24, 2020) as an example, the nominal orbit design result of the phasing stage for the ascent module is given in Table 2. Here, "MCE" refers to the Moon-centered mean equatorial inertial frame and "MJ2000" to the Moon-centered J2000 frame. The total delta- V is 49.7 m/s, which is slightly greater than the total delta- V of the baseline four-impulse scheme. The reason for this is that the maneuver locations in the nominal design are not rigorously the special points. For example, the AOL of the first maneuver is adjusted to satisfy the tracking condition requirement; consequently, it is 10° less than the AOL of the apolune.

Note that the height of the highest mountain on the Moon is about 10 km and the selection of the perilune altitude 15 km is mainly due to the safety consideration and design optimization of the guidance of the powered lunar ascent. Moreover, the apolune altitude 180 km is selected because the orbit altitude of the ascent module at the initial aim point is chosen as 210 km and there is a constraint in the capability of the ascent module (limited propellant). For a detailed discussion on the computation of the initial orbit of the ascent module at insertion, refer to Reference [14].

5.4. Phasing Orbit Design of Orbiter. Before the lunar ascent of the ascent module, the orbiter and reentry capsule assembly needs to adjust its phase angle, orbit altitude, and shape to achieve the prescribed AOL, orbit altitude (200 km), and eccentricity (0) at the prescribed time of the initial aim point. Adjustment of the orbit plane for the orbiter is not required because the orbit plane adjustment is completed by the ascent module, as discussed earlier.

Although the assembly can achieve the desired AOL for the initial aim point at some instant by free flying without maneuvers, it cannot be ensured that the eccentricity will

TABLE 2: Nominal orbit scheme of phasing stage.

Events	Number of revolutions	Perilune altitude (km)	Aposelene altitude (km)	MCE AOL (°)	MJ2000 AOL (°)	Delta-V (m/s)	Tracking stations
Insertion	1	15	180	108.0	140.2		Chinese stations
Δv_1	4	159.5	193.9	278.0	310.1	35.0	Chinese stations
Δv_2	9	160.0	194.5	202.3	234.5	0.9	Foreign stations
Δv_3	15	174.7	210.4	60.0	92.2	6.6	Chinese stations
Δv_4	22	209.5	210.8	290.8	323.2	7.2	Foreign stations
Aim point	25	210.0	210.0	274.296	301.479		Chinese stations

be close to zero simultaneously. Moreover, it is difficult to fully utilize the limited tracking resources when free flying is adopted to achieve the desired AOL; as a result, the subsequent far-range, close-range, and docking operations may have poor ground support. Therefore, the phasing maneuvers of the orbiter (the assembly) are necessary.

The nominal phasing orbit of the orbiter can be obtained by employing a design method similar to that for the phasing orbit design of the ascent module (refer to References [4, 5, 14] for details).

5.5. Design of Initial Aim Point. At the prescribed initial aim point time, the ascent module arrives at the circular lunar orbit of altitude 210 km and is 50 km ahead of the orbiter along track, when the orbiter is flying in a coplanar circular orbit of altitude 200 km, as shown in Figure 8(b). Based on this requirement, the desired relative position and velocity of the ascent module with respect to the orbital frame of the orbiter can be obtained, referring to Table 3. In the table, “x,” “y,” and “z” represent the radial, transverse, and normal directions, respectively, and subscript “r” represents a relative variable.

The design of the location of the aim point in a tracking arc is critical for ensuring that the subsequent far-range, close-range, and docking operations have sufficient ground support. Referring to Figure 12, the 3.5-hour far-range and close-range operations extend over three tracking arcs to ensure that both the aim point and docking point conditions are monitored by the primary ground stations. Moreover, the aim point is at an orbit location approximately 10 min from the ending of the tracking arc. This is to accommodate the maximum possible along-track relative position error at the aim point, which is approximately 6 km from the dispersion analysis of the aim point.

The front of the tracking arc in which the aim point is located can be aligned with the start of the tracking period of the Moon center for the primary ground stations in China to ensure the subsequent far-range, close-range, and docking operations have sufficient ground support. Then, the absolute time of the initial aim point can be determined by a simple addition of 74 min to the start time of the tracking period, referring to Figure 12. Concurrently, the AOL of the initial aim point can be computed from the relative position of the initial aim point in the tracking arc (74 min from the start and 10 min from the end) and the boundary values of the AOL for the tracking arc (69°–302° in Figure 10) [14].

TABLE 3: Nominal relative position and velocity at aim point.

Relative position and velocity components	Desired value
x_r (km)	9.351511
y_r (km)	50.252499
z_r (km)	0
v_{rx} (km/s)	0.000317416
v_{ry} (km/s)	−0.012296498
v_{rz} (km/s)	0

5.6. Dispersion Analysis of Initial Aim Point. During the phasing stage of the RVD operation, the actual orbit deviates from the nominal orbit because of the lunar orbit insertion error of the ascent module, orbit determination error, and orbit control error (maneuver execution error). This may lead to unacceptably large dispersions at the initial aim point.

To reduce the effects of these errors on the initial aim point conditions and maintain the actual orbit close to the nominal orbit, an orbit control strategy needs to be devised. Specifically, before each maneuver, all remaining maneuvers have to be recomputed based on the latest orbit determination data. The orbit control strategy devised for the ascent module is summarized in Table 4. In the table, superscripts “t” and “n” denote the components in the transverse and normal directions, respectively, letter “u” denotes the AOL, and “J” is the weighted function of the relative position and velocity errors at the initial aim point. Taking the first maneuver as an example, the design variables are $[\Delta v_1^t, \Delta v_2^n, u_2, \Delta v_3^t, \Delta v_4^t, u_4]$, and the nominal values of these variables are used as the initial values in computing their accurate values. An iterative differential correction approach is adopted for this computation, targeting the desired relative position and velocity at the initial aim point: $[x_r, y_r, z_r, v_{rx}, v_{ry}, v_{rz}]$ (refer to Table 3).

It is highly desired to only have small errors of the relative position and velocity at the initial aim point with respect to the nominal condition to ensure the success of the remaining rendezvous operations. Numerical analysis can be conducted to assess the dispersions at the initial aim point. Specifically, applying the above orbit control strategy, a Monte Carlo simulation study can be performed for given lunar orbit insertion error, orbit determination error, and orbit control error, and the statistics of the relative position

FIGURE 12: Initial aim point design.

TABLE 4: Orbit control strategy for ascent module.

Maneuvers	Design parameters	Desired aim point condition
First maneuver	$\Delta v_1^t, \Delta v_2^t, u_2, \Delta v_3^t, \Delta v_4^t, u_4$	$\Delta x_r = 0, \Delta y_r = 0, \Delta z_r = 0, \Delta v_{rx} = 0, \Delta v_{ry} = 0, \Delta v_{rz} = 0$
Second maneuver	$\Delta v_2^t, u_2, \Delta v_3^t, u_3, \Delta v_4^t, u_4$	$\Delta x_r = 0, \Delta y_r = 0, \Delta z_r = 0, \Delta v_{rx} = 0, \Delta v_{ry} = 0, \Delta v_{rz} = 0$
Third maneuver	$\Delta v_3^t, u_3, \Delta v_4^t, u_4$	$\Delta x_r = 0, \Delta y_r = 0, \Delta v_{rx} = 0, \Delta v_{ry} = 0;$ $\min J = \Delta x_r + \Delta y_r + k_1 \Delta v_{rx} + k_2 \Delta v_{ry} $, where $k_1 = 1000$ and $k_2 = 1000$
Fourth maneuver	$\Delta v_4^t, u_4$	$\Delta x_r = 0, \Delta y_r = 0$

TABLE 5: Error analysis results of aim point for ascent module.

Statistics	Relative position error (m)			Relative velocity error (m/s)		
	Radial	Transverse	Normal	Radial	Transverse	Normal
μ	38.1	-50.8	5.1	0.1	0.0	0.0
3σ	681.7	6614.7	964.7	1.7	0.9	1.2
Min	-1148.2	-5549.0	-1321.3	-3.6	-1.9	-1.2
Max	1550.2	5543.2	1050.3	6.5	1.6	1.3

and velocity errors at the initial aim point can be obtained. Table 5 summarizes the error analysis results of the initial aim point for the ascent module obtained in the design phase of the Chang'e 5 mission.

An orbit control strategy is also devised for the phasing of the orbiter, and subsequently, a dispersion analysis is conducted for the initial aim point parameters for the orbiter. Based on the dispersion analysis results of the initial aim point, further simulation studies were conducted on the performance of the subsequent far-range and close-range operations, which verified the correctness of the nominal orbit design and the effectiveness of the orbit control strategy design from a numerical simulation perspective.

5.7. Flight Test Results. The Chang'e 5 mission was launched on November 24, 2020. During the flight test, the first unmanned lunar orbit RVD in the world was accomplished with tremendous success. It consisted of several stages, such as the phasing of the orbiter, lunar ascent of the ascent module, phasing of the ascent module, far-range operation, close-

range operation, and docking. In the following, some flight data are provided and compared with the nominal design.

In Table 6, the actual phasing orbit parameters of the ascent module are shown with the nominal orbit parameters.

As can be seen from the table, the flight orbit parameters are consistent with the nominal design. However, during the flight test, the AOL of the third maneuver was increased to increase the duration of the tracking access before the third maneuver. Moreover, the design variables of the fourth maneuver used in flight test were the transverse and radial components of the impulse, instead of the transverse component of the impulse and the AOL of the maneuver used in the nominal design; that is, the AOL of the maneuver was actually fixed during the real flight, which was to ensure that the fourth maneuver would be executed with the tracking arc. This led to a significant increase in the delta- V of the fourth maneuver, which was acceptable in the flight test because there was sufficient propellant remaining in the ascent module.

The delta- V of the second maneuver is greater than the nominal design (coplanar ascent), which is due to the

TABLE 6: Flight orbit of phasing stage (Moon-centered J2000).

Event	Time (UTC)	Perilune altitude (km)	Apolune altitude (km)	AOL (°)	Delta-V (m/s)
Insertion					
Nominal	2020-12-03 15:16	15.0	180.0	140.2	
Flight	2020-12-03 15:16	15.0	188.3	139.8	
Δv_1					
Nominal	2020-12-03 22:04	159.5	193.9	310.1	35.0
Flight	2020-12-03 22:02	153.1	207.9	305.7	34.7
Δv_2					
Nominal	2020-12-04 08:04	160.0	194.5	234.5	0.9
Flight	2020-12-04 07:56	153.4	208.9	205.3	3.9
Δv_3					
Nominal	2020-12-04 19:46	174.7	210.4	92.2	6.6
Flight	2020-12-04 20:06	158.2	218.7	134.8	3.3
Δv_4					
Nominal	2020-12-05 11:54	209.5	210.8	323.2	7.2
Flight	2020-12-05 11:56	209.5	210.6	323.7	18.4
Initial aim point					
Nominal	2020-12-05 18:14	210.0	210.0	301.479	
Flight	2020-12-05 18:14	209.8	210.1	301.362	

TABLE 7: Flight data of aim point error for ascent module.

	Relative position (m)			Relative velocity (m/s)		
	Radial	Transverse	Normal	Radial	Transverse	Normal
Nominal	9351.511	50252.499	0	0.317	-12.296	0
Flight	9272.041	49507.546	-59.200	0.287	-12.169	0.141
Difference	-79.470	-744.953	-59.200	-0.030	0.127	0.141

landing location error. The actual landing site is approximately 11 km away from the nominal landing site; therefore, the ascent module was not in the orbit plane of the orbiter when the ascent module was launched at the prescribed time. This is acceptable as the plane correction delta-V (3.9 m/s) is much less than the plane correction delta-V budget (25 m/s).

The actual errors of the relative position and velocity at the initial aim point in the flight test are listed in Table 7. It can be seen from the comparison of Tables 5 and 7 that the errors at the initial aim point in the actual flight were very small and fulfilled the accuracy requirements for the transition to the far-range (autonomous control) operation.

5.8. Summary. A four-impulse phasing orbit scheme is found to be the best baseline design for the phasing stage of the ascent module in the lunar orbit rendezvous operation. Applying the lunar orbit design method based on tracking analysis, the nominal orbit is obtained by optimization, with consideration of multiple design constraints including both the delta-V and tracking access requirements. The design of the initial aim point, orbit control strategy, and dispersion analysis of the initial aim point are also discussed.

The presented flight data show that the phasing orbit design of the lunar orbit RVD in the Chang'e 5 mission is accurate, and all design requirements are satisfied. The actual delta-V is well below the budget, the tracking condition is good throughout the flight, and the control accuracies of the relative position and velocity at the initial aim point meet the requirement for the transition to the autonomous control operation. As such, the subsequent docking and sample transferring operations were able carried out successfully.

6. Orbit Design of Moon-to-Earth Transfer

After the lunar orbit RVD operation is completed, the orbiter and reentry capsule assembly is flying in a near-circular lunar orbit. After approximately six days, the assembly enters the escape trajectory by performing a few TEI maneuvers and continues to fly in the Moon-to-Earth trajectory, with the end point of the trajectory as the entry point on top of the Earth atmosphere. The optimization of the TEI strategy is a classical orbit design problem in lunar exploration missions and was critical to the accomplishment of the Chang'e 5 mission.

The design of a TEI strategy has been conducted in many studies reported in the literature [20–22]. In the

Chang'e 5 mission design, the optimization algorithm of a two-impulse TEI strategy was studied based on the design results of an improved one-impulse TEI strategy, considering the practical constraints in the engineering design. A three-impulse TEI strategy suitable for large plane changes between the initial lunar orbit plane and the plane of the lunar escape trajectory was also studied. Moreover, in the Chang'e 5 mission design, it was identified as a highly risky scenario that later in the mission, the orbiter's remaining fuel is insufficient for completing the conventional TEI maneuvers, in which case a low-energy Moon-to-Earth transfer orbit was designed as an emergency plan [8, 9].

6.1. Problem Statement. In the Chang'e 5 mission design, it was found that if the one-impulse TEI strategy was adopted, the use of an engine with a large thrust magnitude will be necessary to achieve a short thrust duration and reduce gravity loss, also alleviate the burden of thermal and battery systems. However, the structure of the orbiter, having a much lighter mass than that in the early mission stage, would be prone to dynamic excitation when subjected to a large thrust, which would lead to poor attitude control during the TEI maneuver and high risk of orbit control failure. Therefore, a two-impulse TEI strategy was chosen to be utilized, in which case a short thrust duration was expected for each maneuver when using an engine with a small thrust magnitude.

Refer to Figure 13 for an illustration of two-impulse TEI strategy. The objective of the first maneuver is to enter an elliptical lunar orbit with an orbit period of 8 h or 12 h, starting from a circular lunar orbit. The objective of the second maneuver is similar to that in the one-impulse TEI strategy, namely, to achieve the desired orbit parameters at the Earth reentry point: reentry point altitude (120 km), reentry angle (flight path angle at the reentry point), inclination at the reentry point, and angle between the position vector of the landing site and the reentry trajectory plane (0°).

Intuitively, for the same Moon-to-Earth transfer problem, the directions of both maneuvers in the two-impulse strategy are basically aligned with that of the maneuver in the one-impulse strategy. Moreover, the objective of the first maneuver is to achieve the desired orbit period (phasing), a one-to-one orbital control. For the second maneuver, a four-to-four differential correction procedure is employed to achieve the same objectives as in the one-impulse TEI strategy, namely, the four reentry point parameters mentioned above, with the design parameters being three components of the delta-V vector and the time of the reentry interface.

The following discussion focuses on the one- and two-impulse TEI strategies from an engineering design perspective [8, 9] and addresses the design of time of flight of the Moon-to-Earth transfer, following which some flight data are presented to illustrate the effectiveness of the approach.

6.2. Computation of Lunar Escape Hyperbolic Excess Velocity. The most energy efficient Moon-to-Earth transfer trajectory is of a Hohmann-type. The apogee of a Hohmann-type transfer is located on the orbit of the Moon about the Earth. The difference between the velocity of the

FIGURE 13: Two-impulse TEI strategy illustration.

probe at the apogee of the transfer trajectory and the velocity of the Moon with respect to the Earth yields the hyperbolic excess velocity, $\bar{v}_\infty^+$, which is an important input for TEI strategy design.

The position and velocity of the Moon at the apogee of the transfer trajectory can be retrieved from ephemeris data, and the position vector of the perigee of the transfer trajectory can be considered to be opposite to that of the Moon at the apogee. Given a typical time of flight of 4–5 days for the Moon-to-Earth transfer, the Lambert algorithm can be employed to compute the delta-V vector at the start point (apogee) of the transfer and obtain the hyperbolic excess velocity, $\bar{v}_\infty^+$.

A more sophisticated algorithm can be employed to compute the lunar escape hyperbolic excess velocity, referring to Reference [21].

6.3. Optimization of One-Impulse TEI Strategy. Referring to Figure 14, the initial lunar orbit parameters are known, with orbital angular momentum being $\bar{h}_0$; the position vector of the intersection point of the Moon-to-Earth transfer trajectory and the lunar orbit is denoted $\bar{r}_1$, and the delta-V vector of the TEI maneuver is denoted $\Delta\bar{v}_1$. Moreover, in the following discussions, subscript “0” represents the initial lunar orbit parameters, subscript “f” refers to the transfer trajectory parameters, and subscript “1” represents the maneuver parameters.

First, the orbit angular momentum, $\bar{h}_f$, of the lunar escape trajectory is calculated. There are infinitely many eligible transfer trajectory planes about $\bar{v}_\infty^+$ because $\bar{v}_\infty^+$ lies in the plane of the Moon-to-Earth transfer trajectory, i.e., there are infinitely many solutions for $\bar{h}_f$. To determine the optimal $\bar{h}_f$ solution corresponding to the minimum TEI delta-V, the initial value of $\bar{h}_f$ can be computed as follows:

$$\bar{h}_f^0 = \frac{(\bar{v}_\infty^+ \times \bar{h}_0) \times \bar{v}_\infty^+}{|(\bar{v}_\infty^+ \times \bar{h}_0) \times \bar{v}_\infty^+|}. \quad (27)$$

Referring to Figure 14, $\bar{h}_f^0$, $\bar{h}_0$, and $\bar{v}_\infty^+$ are all on the same plane.

FIGURE 14: One-impulse TEI strategy.

To find the optimal $\vec{h}_f$, $\vec{h}_f^0$ is rotated about $\vec{v}_\infty^+$ continuously by varying the rotation angle, θ , from 0° to 360° (step size $\Delta\theta = 1^\circ$), and the new direction of the angular momentum of the lunar escape trajectory is expressed as follows:

$$\hat{h}_f^* = \cos \theta \cdot \hat{h}_f^0 + \sin \theta \cdot (\hat{v}_\infty^+ \times \hat{h}_f^0), \quad (28)$$

where superscript “ $\wedge$ ” denotes a unit vector. Note that it is a rule of thumb to use a step length $\Delta\theta = 1^\circ$, which would introduce a numeric error that is no greater than 0.1 m/s in computing the optimal maneuver in the following two-body model analysis. This is sufficiently accurate and will not affect the final design results because the precise value of the optimal maneuver is obtained later via differential correction and optimization, referring to the subsequent discussion.

The following computation procedure follows the development in References [8, 9, 21]. For each $\hat{h}_f^*$ obtained above, the TEI delta-V can be computed as follows:

- (1) The lunar orbit parameters are computed as follows:

$$\begin{aligned} \hat{r}_1 &= \text{sign} \left(\hat{h}_0 \cdot \hat{v}_\infty^+ \right) \frac{\hat{h}_0 \times \hat{h}_f^*}{|\hat{h}_0 \times \hat{h}_f^*|}, \\ f_{10} &= \text{sign} \left[\hat{h}_0 \cdot (\hat{r}_{p0} \times \hat{r}_1) \right] \cdot \arccos (\hat{r}_{p0} \cdot \hat{r}_1), \\ r_1 &= \frac{a_0(1 - e_0^2)}{1 + e_0 \cos f_{10}}, \\ p_0 &= a_0(1 - e_0^2), \end{aligned} \quad (29)$$

where f_{10} denotes the true anomaly of the maneuver point on the lunar orbit and $\hat{r}_{p0}$ denotes the direction of the position vector of the lunar orbit perilune.

- (2) The lunar escape trajectory parameters are calculated as follows:

$$a_f = -\frac{\mu_m}{v_\infty^2}, \quad (30)$$

$$\Delta f = f_\infty^+ - f_{1f} = \arccos (\hat{r}_1 \cdot \hat{v}_\infty^+),$$

where μ_m denotes the lunar gravitational constant and Δf denotes the difference between the true anomaly, f_∞^+ , of the asymptote of the lunar escape trajectory and the true anomaly, f_{1f} , of the maneuver point on the lunar escape trajectory.

Since $\cos f_\infty^+ = -1/e_f$,

$$\cos f_{1f} = \cos (f_\infty^+ - \Delta f) = -\frac{1}{e_f} \cos \Delta f + \frac{\sqrt{e_f^2 - 1}}{e_f} \sin \Delta f. \quad (31)$$

The orbit equation yields the following:

$$r_1 = \frac{a_f(1 - e_f^2)}{1 + e_f \cos f_{1f}}. \quad (32)$$

Solving the above two equations simultaneously for e_f and f_{1f} yields the following:

$$\begin{aligned} e_f &= \frac{\sqrt{\sin^2 \Delta f + 2\kappa^2 + 2\kappa(1 - \cos \Delta f) + \sin \Delta f \sqrt{\sin^2 \Delta f + 4\kappa(1 - \cos \Delta f)}}}{\sqrt{2\kappa}}, \\ f_{1f} &= \arccos \left(-\frac{1}{e_f} \cos \Delta f + \frac{\sqrt{e_f^2 - 1}}{e_f} \sin \Delta f \right), \end{aligned} \quad (33)$$

where $\kappa = -a_f/r_1$.

The deflection angle and the semilatus rectum can be computed as follows once e_f is obtained:

$$\begin{aligned}\delta &= 2 \arcsin \frac{1}{e_f}, \\ p_f &= a_f (1 - e_f^2).\end{aligned}\quad (34)$$

(3) The maneuver delta-V, $\Delta \bar{v}_1$, is calculated as follows:

$$\begin{aligned}\hat{v}_{\infty}^- &= \cos \delta \cdot \hat{v}_{\infty}^+ + \sin \delta \cdot \left(\hat{v}_{\infty}^+ \times \hat{h}_f^* \right), \\ \hat{r}_{pf} &= \frac{\hat{v}_{\infty}^- - \hat{v}_{\infty}^+}{\left| \hat{v}_{\infty}^- - \hat{v}_{\infty}^+ \right|}, \\ \hat{v}_{pf} &= \frac{\hat{v}_{\infty}^- + \hat{v}_{\infty}^+}{\left| \hat{v}_{\infty}^- + \hat{v}_{\infty}^+ \right|}, \\ \bar{v}_1^+ &= \sqrt{\frac{\mu_m}{p_f}} \left[-\sin f_{1f} \cdot \hat{r}_{pf} + (e_f + \cos f_{1f}) \cdot \hat{v}_{pf} \right], \\ \bar{v}_1^- &= \sqrt{\frac{\mu_m}{p_0}} \left[-\sin f_{10} \cdot \hat{r}_{p0} + (e_0 + \cos f_{10}) \cdot \hat{v}_{p0} \right], \\ \Delta \bar{v}_1 &= \bar{v}_1^+ - \bar{v}_1^-.\end{aligned}\quad (35)$$

Here, $\hat{r}_{pf}$ and $\hat{v}_{pf}$ denote the perilune position direction and perilune velocity direction of the lunar escape trajectory, respectively, $\bar{v}_1^-$ and $\bar{v}_1^+$ denote the velocity of the probe before and after the maneuver, respectively, and $\hat{v}_{p0}$ denotes the direction of the perilune velocity of the lunar parking orbit.

The above procedure is repeated for θ (0° – 360°), and the minimum delta-V is obtained, which corresponds to the optimal maneuver, $\Delta \bar{v}_1$. The optimal AOL of the TEI maneuver is as follows:

$$u_{10} = \omega_0 + f_{10}. \quad (36)$$

After the optimal AOL, u_{10} , and $\Delta \bar{v}_1$ of the TEI maneuver are obtained, the previously mentioned four-to-four differential correction is applied to determine the precise numerical solution of the one-impulse TEI strategy using a high-precision orbital dynamic model. Because the four-to-four differential correction also yields the precise lunar escape $\bar{v}_{\infty}^+$, the above optimization procedure using a two-body model can be repeated to yield improved optimal AOL u_{10} and $\Delta \bar{v}_1$.

In the optimization design of the one-impulse TEI strategy, application of an additional phasing maneuver a few days before the TEI is necessary to ensure that the probe achieves the optimal AOL, u_{10} , at the desired TEI maneuver

time. In addition, note that tracking condition analysis is necessary for the practical application of the TEI strategies. The preceding tracking analysis-based lunar orbit design method is employed. Specifically, the period during which the Moon is visible to the ground stations (the tracking period) is obtained, and the time instants of the starting and ending of the tracking period can be used as a natural time reference in planning the maneuvers.

The computation procedure of the preceding one-impulse TEI strategy is summarized in Reference [8].

6.4. Two-Impulse TEI Strategy. The optimal AOL of the TEI maneuver, u_{10} , and the delta-V vector, $\Delta \bar{v}_1$, are obtained from the one-impulse TEI design, using which the optimal two-impulse TEI strategy can be determined. The computation procedure of the two-impulse strategy is as follows:

- (1) The time of the second TEI maneuver is chosen as the same as the TEI maneuver time in the one-impulse strategy; i.e., it is the start time of the tracking period on the day of the TEI plus 30 min. Moreover, the initial value of the delta-V vector of the second TEI maneuver is chosen as one half of the delta-V vector in the one-impulse strategy, i.e., $\Delta \bar{v}_1 / 2$.
- (2) Based on the results of the tracking analysis, the first TEI maneuver time is after the start time of the tracking period on the day before the day of the second TEI maneuver. The maneuver is executed when the orbiter is at the orbital position of the AOL, $u_1 = u_{10}$. Moreover, the initial value of delta-V is taken as $\Delta v_1 / 2$, with its direction (azimuth α and declination δ) being the same as $\Delta \bar{v}_1$. The objective of the first maneuver is to achieve AOL $u_2 = u_{10}$ at the chosen time of the second TEI maneuver (phasing).
- (3) Precise numerical solutions of the two maneuvers are calculated using the high-precision orbital dynamic model, in which the first maneuver is computed with a one-to-one differential correction and the second maneuver with a four-to-four differential correction, same as in the one-impulse strategy.
- (4) If necessary, numerical optimization can be conducted using the initial values of u_1 , α , δ , and u_2 given above to reduce the total delta-V as follows:

$$\left(\sum \Delta V \right)_{\min} = f(u_1 \quad \alpha \quad \delta \quad u_2). \quad (37)$$

6.5. Trade Study of Time of Flight of Moon-to-Earth Transfer. In the mission design of the LSR mission, the preceding TEI maneuver algorithm was used for extensively searching candidate TEI dates, in which the time of flight of the Moon-to-Earth transfer was initially chosen as approximately five days (112 h). When the launch of the Chang'e 5 mission was postponed to 2020, only one launch window in November (November 25, 2020) was found, which was highly undesirable for the launch center. The launch center preferred at

TABLE 8: Time of flight and reentry flight range.

Moon-to-Earth time of flight (h)	TEI delta-V (m/s)	Reentry point inclination (°)	Reentry flight range (km)	Comment
112	850	45.0	7218	Initial design
112+24	850	45.0	7986	
112-24	847	45.0	6619	

TABLE 9: Reentry point parameter error (Earth-centered fixed frame).

	Nominal design	Flight data	Difference	Control accuracy requirement
Reentry time (UTC)	2020-12-16 17:32:35.126	2020-12-16 17:32:59.269	24.143 sec	—
Reentry point altitude (km)	120	120.000	0.000	—
Flight path angle (°)	−5.75	−5.748	0.002	0.2
Reentry point inclination (°)	45	44.99987	−0.00013	0.1

least two successive launch windows to reduce the launch risk.

A close examination showed that the date of November 24, 2020 was eliminated from the candidate launch windows because the corresponding reentry flight range of 7218 km was not satisfying the constraint on the reentry range, 5600 km–7100 km. Thus, it was proposed by the author to vary the time of flight of the Moon-to-Earth to adjust the reentry flight range, as summarized in the analysis results in Table 8.

As can be seen from the table, the time of flight of 112 – 24 = 88 h should be selected to satisfy the design constraint on the reentry flight range. This design revision was adopted, and the date of November 24, 2020 became the actual launch date of the Chang'e 5 mission.

6.6. Flight Test Results. In Table 9, the nominal orbit parameters of the reentry point are compared with their actual values in the flight test of the Chang'e 5 mission. The control errors for these parameters are quite small, much less than the required control accuracy, which is an indication that the nominal TEI orbit design for the Chang'e 5 mission was accurate. Actually, the last trajectory correction maneuver during the Moon-to-Earth transfer was cancelled because of the predicted high control precision of the reentry point parameters. Note that during the flight test, the targeted nominal value of the flight path angle was adjusted from -5.8° to -5.75° based on the evaluation of the predicted reentry trajectory.

6.7. Summary. The optimization algorithm of the two-impulse TEI strategy developed in the Chang'e 5 mission design is discussed in detail considering the engineering design constraints. The actual flight data are compared with the nominal design to demonstrate the validity and correctness of the TEI orbit design.

7. Concluding Remarks

The three key orbit design technologies employed in the Chang'e 5 mission are identified and discussed in detail. They are the orbit design for precision lunar landing and

inclination optimization, orbit design for lunar orbit RVD, and TEI orbit design for the Moon-to-Earth transfer. The tracking analysis-based lunar orbit design method is employed in all three orbit designs and is essential for selecting the maneuver times and satisfying the tracking access condition requirements. Actual flight data are provided to demonstrate the precision and effectiveness of the three orbit design technologies, which are the most critical orbit design elements supporting the overall mission success.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

Disclosure

A preliminary version of this paper was presented at the 2021 AAS/AIAA Astrodynamics Specialist Conference, and the authors are thankful to the audience for its motivating comments.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Z. Wang is the principal investigator of the three key orbit design technologies discussed in the paper. Z. Meng is the supervisor of the investigation efforts and offered many valuable helps and suggestions. S. Gao assisted in the investigation with numerical simulation, tracking, and lighting condition analysis. J. Peng provided important inputs from the spacecraft system design and helpful comments during the investigation.

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