AAAS
Space: Science & Technology
Volume 2021, Article ID 9801421, 7 pages
https://doi.org/10.34133/2021/9801421

Space: Science & Technology
A SCIENCE PARTNER JOURNAL

Research Article

A New Recursive Composite Adaptive Controller for Robot Manipulators

Jianfei Li,[1,2] Yaobing Wang,[2] Zhiyong Liu,[1] Xin Jing,[3] and Chengwei Hu[2]

[1]*The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China*
[2]*Beijing Key Laboratory of Intelligent Space Robotic System Technology and Applications, Beijing Institute of Spacecraft System Engineering, Beijing 100094, China*
[3]*State Key Laboratory of Robotics and System, Harbin Institute of Technology, Harbin 150001, China*

Correspondence should be addressed to Yaobing Wang; iamwyb@163.com and Xin Jing; xin.jing@hit.edu.cn

Received 29 June 2021; Accepted 26 September 2021; Published 27 October 2021

In this paper, a new recursive implementation of composite adaptive control for robot manipulators is proposed. We investigate the recursive composite adaptive algorithm and prove the stability directly based on the Newton-Euler equations in matrix form, which, to our knowledge, is the first result on this point in the literature. The proposed algorithm has an amount of computation $O(n)$, which is less than any existing similar algorithms and can satisfy the computation need of the complicated multidegree manipulators. The manipulator of the Chinese Space Station is employed as a simulation example, and the results verify the effectiveness of this proposed recursive algorithm.

1. Introduction

In the process of the construction and routine maintenance of the Chinese Space Station, the manipulator of the Chinese Space Station plays a significantly important role that can accomplish some key tasks, such as transposition docking, daily maintenance, and auxiliary extravehicular activities [1, 2]. The high accuracy and dynamic performance of the manipulator are the necessary conditions for the successful completion of these tasks, which can often be maintained by controls that are designed based on the dynamics model. But in the practice situations, it is usually unrealistic to obtain all the inertia parameters precisely. An adaptive control scheme is one approach that can overcome this problem in spite of large parameter uncertainties.

Adaptive control can ensure the convergence of tracking control even if the system has uncertain or slowly changing parameters. In general, this scheme can be divided into two classes named the direct adaptation and the indirect adaptation according to the signal that drives the parameter update law. In the first category, the parameter update is driven by the tracking errors. While in the second category, the parameters are modified according to the prediction errors, usually of the filtered joint torques. Adaptive control based on tracking errors usually can guarantee a global tracking convergence; however, the converge of estimated parameters has more stringent conditions. In comparison, the indirect adaptive control has a faster parameter convergence speed, but it is generally difficult to obtain the stability of the tracking errors. Combining the two methods, the well-known composite adaptive controller has the advantages of both, in which the parameter adaptation is driven by both tracking errors and prediction errors [3, 4].

Some more recent results on adaptive manipulator control focus on dealing with the control of flexible robots [5, 6]. In some applications of coordinated manipulators, some adaptive control methods are also attractive that behave with perfect performance [7–10]. In order to achieve higher performance, adaptive control schemes are usually combined with other control methods, such as robust control methods [11, 12], and neural networks [13, 14].

However, computational complexity of these adaptive control methods is a main limitation in the practical robot manipulators, particularly for the case with high degree of freedom. It is straightforward for using Newton-Euler formulation for the adaptive controller based on the computed torque method, while it seems hard to seek an efficient approach for the adaptive controller based on the passivity theory, mainly for the emergency of the reference velocity in the Coriolis and centrifugal matrix [15]. But there were still some works concerning this problem. Using the spatial vector notation [16], Niemeyer and Slotine proposed a recursive scheme for the controller of [15], and the computational load is $O(n)$ [17]. Huo and Gao proposed a much simpler scheme, via defining a new Coriolis and centrifugal matrix which satisfies the skew-symmetric property [18]. Zhu proposed the VDC control which realized direct adaptive control directly based on the Newton-Euler dynamics [19].

The studies in [17–19] considered only the recursive form of direct adaptive control; nevertheless, to the best of our knowledge, only a few works paid attention to the recursive execution of the indirect adaptive or the composite adaptive controller, probably because the use of the prediction error expressed by the regression matrix makes it very difficult to reduce the order of the computational complexity. In this regard, Wang proposed a recursive scheme of the composite adaptive controller using the Spatial Notation, with a computational complexity $O(n^2)$, which reduces the order of computational complexity compared with the non-recursive composite adaptive controller [20].

In this paper, we design a recursive composite adaptive controller and prove the stability directly based on Newton-Euler equations in matrix form. The filtered first joint torque is used to obtain a linear relationship of dynamics. Deriving the algorithm into a recursive form, we successfully reduce the computational load to $O(n)$.

The rest of this paper is organized as follows: In Section 2, the composite adaptive controller proposed by Slotine and Li in [4] is revisited, followed by a description of the Newton-Euler formulation which is based on the matrices. Section 3 presents the recursive composite adaptive controller and the stability analysis. Simulation results in Section 4 demonstrate the effectiveness of this proposed recursive scheme. Finally, the discussion and conclusion are offered in Section 5.

2. Preliminaries

2.1. Composite Adaptive Controller. Firstly, it is necessary to revisit the composite adaptive controller proposed by Slotine and Li in [4], which is helpful to understand our recursive composite adaptive control method. The dynamic model of an n-dof robot manipulator neglecting the frictional forces can be expressed as follows:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau, \tag{1}$$

where $\mathbf{q} \in \mathbb{R}^{n\times1}$ is the position vector in the joint space, $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the inertia matrix; $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \in \mathbb{R}^{n\times1}$ is the Coriolis and centrifugal force, $\mathbf{G}(\mathbf{q}) \in \mathbb{R}^{n\times1}$ is the gravitational force, and $\tau \in \mathbb{R}^{n\times1}$ is the joint torque. The adaptive control problem is as follows: given the desired joint position, velocity, and acceleration $\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d$, and that the value of the joint position $\mathbf{q}$ can be measured from the encoder, $\dot{\mathbf{q}}$ can be obtained by numerical differentiation; the object of the adaptive controller is that the tracking errors of all the joints converge to zero, even if some dynamic parameters are unknown.

The control law and adaptation law in [21] can be presented as follows:

$$\tau = \hat{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}}_r + \hat{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \hat{\mathbf{G}}(\mathbf{q}) - \mathbf{K}_D\mathbf{s}, \tag{2}$$

$$\dot{\hat{\theta}} = -\Gamma^{-1}\mathbf{Y}^{\mathrm{T}}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\mathbf{s}, \tag{3}$$

where $\dot{\mathbf{q}}_r = \dot{\mathbf{q}}_d - \Lambda\tilde{\mathbf{q}}$, $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$, $\mathbf{s} = \dot{\tilde{\mathbf{q}}} + \Lambda\tilde{\mathbf{q}} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r$, Λ, and $\mathbf{K}_D$ are symmetric positive definite matrices; $\theta \in \mathbb{R}^{m\times1}$ is the vector containing the unknown dynamic parameters; and $\hat{\theta}$ is its estimate, $\tilde{\theta} = \hat{\theta}(t) - \theta$. The matrix $\mathbf{Y}$ is defined as

$$\tilde{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}}_r + \tilde{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \tilde{\mathbf{G}}(\mathbf{q}) = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\tilde{\mathbf{a}}, \tag{4}$$

where $\tilde{\mathbf{M}} = \hat{\mathbf{M}} - \mathbf{M}$, $\tilde{\mathbf{C}} = \hat{\mathbf{C}} - \mathbf{C}$, and $\tilde{\mathbf{G}} = \hat{\mathbf{G}} - \mathbf{G}$.

The adaptive update law of (3) is driven by the tracking errors of the joint tracking motion; on the other hand, prediction errors on filtered joint torques are requested to add into the parameter estimates in the composite adaptive controller [4]. Before giving the "composite adaptation law," we need to resolve the filtered joint torque to avoid the measurement of the joint acceleration, filtering (1) with a first-order filer $\lambda/(p + \lambda)$ yields

$$\tau_f = \int_0^t w(t - r)[\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q})]dr = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}, t)\theta, \tag{5}$$

where p is the Laplace variable, $\lambda > 0$ is the filter parameter, and $w(t)$ is the impulse response of the filter. $\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}, t)$ is the filtered regressor matrix. So the prediction errors $\mathbf{e}$ on filtered joint torques can be obtained as follows:

$$\mathbf{e} = \hat{\tau}_f - \tau_f = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})\tilde{\theta}. \tag{6}$$

The composite adaptation update law can be described as

$$\dot{\hat{\theta}} = -\mathbf{P}\left[\mathbf{Y}^{\mathrm{T}}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\mathbf{s} + \mathbf{W}^T\mathbf{e}\right], \tag{7}$$

where $\mathbf{P}$ is the adaptation gain, which is a constant matrix in the case of the gradient adaptation.

To sum up, Equations (2) and (7) constitute the composite adaptive controller [3, 4].

2.2. Matrix-Based Newton-Euler Dynamics. In this part, the classic Recursive Newton-Euler Algorithm (RNEA) is rewritten by the form of general matrices. The frames of the adjacent links are represented in Figure 1. All the joints are rotating with the z axis. Define ${}^{i}\mathbf{v}_i \in \mathbb{R}^{3\times1}$ and ${}^{i}\omega_i \in \mathbb{R}^{3\times1}$ as the linear velocity and the angular velocity

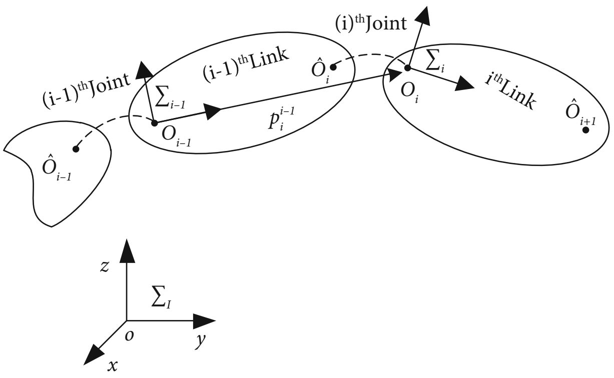

Figure 1: The coordinate frame relationship between the links.

expressed in the frame $\sum_i$, respectively. Define the general velocity expressed in its own frame, as follows [19]:

$$ {}^{i}\mathbf{V}_i = \left({}^{i}\mathbf{v}_i^{T} \, {}^{i}\omega_i^{T} \right)^{T} \in \mathbb{R}^{6\times1}. \tag{8}$$

The general velocity transferring formulation can be obtained by the RNEA; for convenience, the presuper i is omitted, which yields

$$\mathbf{V}_i = {}^{i}\mathbf{T}_{i-1}\mathbf{V}_{i-1} + \mathbf{z}_6\dot{q}_i, \tag{9}$$

where $\mathbf{z}_6 = [0\,0\,0\,0\,0\,1]^T$ and ${}^{i}\mathbf{T}_{i-1}$ is the transform matrix of the general velocity, which is defined as follows:

$$ {}^{i}\mathbf{T}_{i-1} = \begin{bmatrix} {}^{i}\mathbf{C}_{i-1} & -{}^{i}\mathbf{C}_{i-1}\left({}^{i\text{-}1}\mathbf{p}_i^{i\text{-}1}\right)^{\times} \\ \mathbf{O}_{3\times3} & {}^{i}\mathbf{C}_{i-1} \end{bmatrix}, \tag{10}$$

where ${}^{i}\mathbf{C}_{i-1}$ is the rotational matrix and ${}^{i-1}\mathbf{p}_i^{i-1}$ is the origin of frame $\sum_i$ expressed in the frame $\sum_{i-1}$; let us define $(\cdot)^{\times}$ as the cross-product matrix of the vector $(\cdot)$.

The general acceleration transferring formulation is expressed as follows:

$$\dot{\mathbf{V}}_i = {}^{i}\dot{\mathbf{T}}_{i-1}\mathbf{V}_{i-1} + {}^{i}\mathbf{T}_{i-1}\dot{\mathbf{V}}_{i-1} + \mathbf{z}_6\ddot{q}_i, \tag{11}$$

where $\dot{\mathbf{V}} = d\mathbf{V}/dt$,

$$ {}^{i}\dot{\mathbf{T}}_{i-1} = \begin{bmatrix} -\left({}^{i}\omega_i^{i\text{-}1}\right)^{\times}{}^{i}\mathbf{C}_{i\text{-}1} & \left({}^{i}\omega_i^{i\text{-}1}\right)^{\times}{}^{i}\mathbf{C}_{i\text{-}1}\left({}^{i\text{-}1}\mathbf{p}_i^{i\text{-}1}\right)^{\times} \\ \mathbf{O}_3 & -\left({}^{i}\omega_i^{i\text{-}1}\right)^{\times}{}^{i}\mathbf{C}_{i\text{-}1} \end{bmatrix}. \tag{12}$$

For the link i, the dynamic equations expressed in frame $\sum_i$ can be written as

$$\mathbf{M}_i \cdot \dot{\mathbf{V}}_i + \mathbf{C}_i \cdot \mathbf{V}_i + \mathbf{G}_i = \mathbf{F}_i^{*} = \mathbf{Y}_i\theta_i, \tag{13}$$

where $\mathbf{F}_i^{*}$ is the total force (torque) acting on the link i, $\mathbf{Y}_i \in \mathbb{R}^{6\times13}$ is the regressor matrix of link i (refer to Zhu's book [19] for the definition of $\mathbf{Y}_i$), and $\boldsymbol{\theta}_i \in \mathbb{R}^{13\times1}$ expresses the inertial parameters of link i, where $\theta_{i1} = m_i$, $\theta_{i2} = m_i\mathbf{p}_{ci}^{i}(1)$, $\theta_{i3} = m_i\mathbf{p}_{ci}^{i}(2)$, $\theta_{i4} = m_i\mathbf{p}_{ci}^{i}(3)$, $\theta_{i5} = m_i\mathbf{p}_{ci}^{i}(1)^2$, $\theta_{i6} = m_i\mathbf{p}_{ci}^{i}(2)^2$, $\theta_{i7} = m_i\mathbf{p}_{ci}^{i}(3)^2$, $\theta_{i8} = m_i\mathbf{p}_{ci}^{i}(1) * \mathbf{p}_{ci}^{i}(2) - \mathbf{I}_{ci}^{i}(1,2)$, $\theta_{i9} = m_i\mathbf{p}_{ci}^{i}(1) * \mathbf{p}_{ci}^{i}(3) - \mathbf{I}_{ci}^{i}(1,3)$, $\theta_{i10} = m_i\mathbf{p}_{ci}^{i}(2) * \mathbf{p}_{ci}^{i}(3) - \mathbf{I}_{ci}^{i}(2,3)$, $\theta_{i11} = \mathbf{I}_{ci}^{i}(1,1)$, $\theta_{i12} = \mathbf{I}_{ci}^{i}(2,2)$, and $\theta_{i13} = \mathbf{I}_{ci}^{i}(3,3)$ [19]. The inertial matrix, the Coriolis and centrifugal force matrix, and the gravitational matrix are as follows:

$$\begin{aligned} \mathbf{M}_i &= \begin{bmatrix} m_i\mathbf{I}_{3\times3} & -m_i\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times} \\ m_i\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times} & {}^{i}\mathbf{I}_{ci} - m_i\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times}\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times} \end{bmatrix}, \\ \mathbf{C}_i &= \begin{bmatrix} m_i(\omega_i)^{\times} & -m_i(\omega_i)^{\times}\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times} \\ m_i\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times}(\omega_i)^{\times} & (\omega_i)^{\times i}\mathbf{I}_{ci} + {}^{i}\mathbf{I}_{ci}(\omega_i)^{\times} - m_i\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times}(\omega_i)^{\times}\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times} \end{bmatrix}, \\ \mathbf{G}_i &= \begin{bmatrix} m_i{}^{I}\mathbf{C}_i\mathbf{g} \\ m_i\left({}^{i}\mathbf{p}_{ci}^{i}\right)^{\times I}\mathbf{C}_i\mathbf{g} \end{bmatrix}, \end{aligned} \tag{14}$$

where m_i is the mass of the link, ${}^{i}\mathbf{p}_{ci}^{i}$ is the position of the center of mass of link i expressed in frame $\sum_i$, and ${}^{i}\mathbf{I}_{ci}$ is the inertia parameters of the link i. ${}^{I}\mathbf{C}_i$ is the rotational matrix of the frame of link i respect with the inertial frame. $\mathbf{g}$ is gravitational acceleration.

The force transferring formulation is expressed as

$$\mathbf{F}_i = {}^{i+1}\mathbf{T}_i^{T}\mathbf{F}_{i+1} + \mathbf{F}_i^{*}. \tag{15}$$

The joint torques can be resolved as follows:

$$\tau_i = \mathbf{z}_6{}^{T}\mathbf{F}_i. \tag{16}$$

To sum up, Equations (9), (11), (15), and (16) are the matrix formulations of the classic RNEA; the initial general velocity, the initial general acceleration, and the general force acting on the end-effector are set to be

$$\mathbf{V}_0 = \mathbf{O}_{6\times1}, \dot{\mathbf{V}}_0 = \mathbf{O}_{6\times1}, \mathbf{F}_{n+1} = -\mathbf{F}_e. \tag{17}$$

3. Recursive Composite Adaptive Control

Based on the formulations in Section 2, the recursive composite adaptive controller is given in this section.

The reference velocity and acceleration of every link can be obtained by Equation (18); the forward recursive equations need to be propagated from the robot base to the tip, for $i = 1, 2, \cdots n$:

$$\begin{aligned} \mathbf{V}_{ri} &= {}^{i}\mathbf{T}_{i-1}\mathbf{V}_{r,i-1} + \mathbf{z}_6\dot{q}_{ri}, \mathbf{V}_{r0} = O_{6\times1}, \\ \dot{\mathbf{V}}_{ri} &= {}^{i}\mathbf{T}_{i-1}\dot{\mathbf{V}}_{r,i-1} + {}^{i}\dot{\mathbf{T}}_{i-1}\mathbf{V}_{r,i-1} + \mathbf{z}_6\ddot{q}_{ri}, \dot{\mathbf{V}}_{r0} = O_{6\times1}. \end{aligned} \tag{18}$$

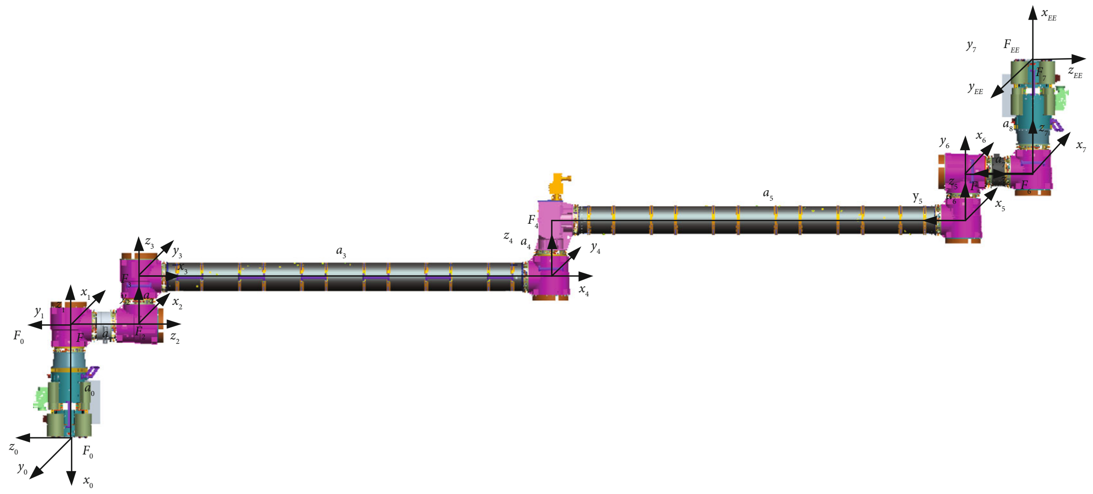

Figure 2: The coordinate frames of the manipulator.

The following backward recursive equations need to be propagated from the robot tip to the base, for $i = n, n-1, \ldots 1$:

$$\mathbf{F}_{ri} = \left({}^{i+1}\mathbf{T}_i\right)^T \mathbf{F}_{r,i+1} + \hat{\mathbf{M}}_i \cdot \dot{\mathbf{V}}_{ri} + \hat{\mathbf{C}}_i \cdot \mathbf{V}_{ri} + \hat{\mathbf{G}}_i + \mathbf{K}_{Di}(\mathbf{V}_{ri} - \mathbf{V}_i), \mathbf{F}_{r,n+1} = O_{6\times 1}, \tag{19a}$$

or

$$\mathbf{F}_{ri} = \left({}^{i+1}\mathbf{T}_i\right)^T \mathbf{F}_{r,i+1} + \mathbf{Y}_{ri}\hat{\theta}_i + \mathbf{K}_{Di}(\mathbf{V}_{ri} - \mathbf{V}_i), \mathbf{F}_{r,n+1} = O_{6\times 1}, \tag{19b}$$

where $\hat{\mathbf{M}}_i, \hat{\mathbf{C}}_i, \hat{\mathbf{G}}_i, \hat{\theta}_i$ are the estimated values of $\mathbf{M}_i, \mathbf{C}_i, \mathbf{G}_i, \theta_i$, and $\mathbf{K}_{Di} \in \mathbb{R}^{6\times 6}$ is the positive definite symmetric feedback gain matrix.

Then, the control torque of every joint can be resolved by Equation (20), as follows:

$$\tau_i = \mathbf{z}_6{}^T \mathbf{F}_{ri}. \tag{20}$$

The above control law contains many estimated values because of the unknown inertial parameters; meanwhile, the parameter adaptive law is also needed. The torque of the first joint is given as Equation (21), and the computational load is $O(n)$. In fact, the first joint is located in the innermost position; it contains all the parameter information of every link. Using τ_1 in the composite adaptive law, which can utilize most of the response information,

$$\tau_1 = \mathbf{z}_6{}^T \sum_{j=1}^{n} {}^j\mathbf{T}_1^T \left(\mathbf{M}_j \cdot \dot{\mathbf{V}}_j + \mathbf{C}_j \cdot \mathbf{V}_j + \mathbf{G}_j\right). \tag{21}$$

Table 1: Physical parameters of the manipulator.

i-th link	m_i (kg)	$\mathbf{I}_{ci}^i$ (kg·m^2)	$\mathbf{p}_{ci}^i$ (m)	$\mathbf{p}_i^{i-1}$ (m)
1	50	diag[2 2 200]	[0 -0.1 0]	[0 -a1 0]
2	50	diag[2 2 200]	[0 0.1 0]	[0 a2 0]
3	100	diag[5 300 500]	[2 0 0]	[a3 0 0]
4	100	diag[5 300 500]	[2 0 0.5]	[a5 0 a4]
5	50	diag[2 2 200]	[0 0 0.5]	[0 0 a6]
6	50	diag[2 2 200]	[0 0 0.5]	[0 0 a7]
7 and load	100	diag[2 2 200]	[0 0.5 1]	[0 0 a8]

Equation (21) needs to measure the joint accelerations; utilizing a first-order low-pass filter, the filtered torque can be resolved as

$$\begin{aligned} y &= \int_0^t w(t-r)\tau_1 dr \\ &= \int_0^t w(t-r)\mathbf{z}_6{}^T \sum_{j=1}^{n} {}^j\mathbf{T}_1^T \left(\mathbf{M}_j \cdot \dot{\mathbf{V}}_j + \mathbf{C}_j \cdot \mathbf{V}_j + \mathbf{G}_j\right) dr \\ &= \mathbf{z}_6{}^T \sum_{j=1}^{n} \left[\begin{array}{c} w(0){}^j\mathbf{T}_1^T \mathbf{M}_j \cdot \mathbf{V}_j - w(t){}^j\mathbf{T}_1^T \mathbf{M}_j \cdot \mathbf{V}_j(0) \\ + \int_0^t w(t-r)\left({}^j\dot{\mathbf{T}}_1^T \mathbf{M}_j \mathbf{V}_j + {}^j\mathbf{T}_1^T \mathbf{C}_j \cdot \mathbf{V}_j + {}^j\mathbf{T}_1^T \mathbf{G}_j\right) dr \end{array} \right] \\ &= \sum_{j=1}^{n} \mathbf{W}_j \theta_j. \end{aligned} \tag{22}$$

So the linear relationship of dynamics is obtained. The adaptive update law of every link can be resolved by the following equation, which is similar with Equation (7).

$$\dot{\hat{\theta}}_i = -\mathbf{P}_i \left[\mathbf{Y}_{ri}{}^T \mathbf{s}_i + \gamma \mathbf{W}_i^T \sum_{j=1}^{n} \mathbf{e}_j \right], \tag{23}$$

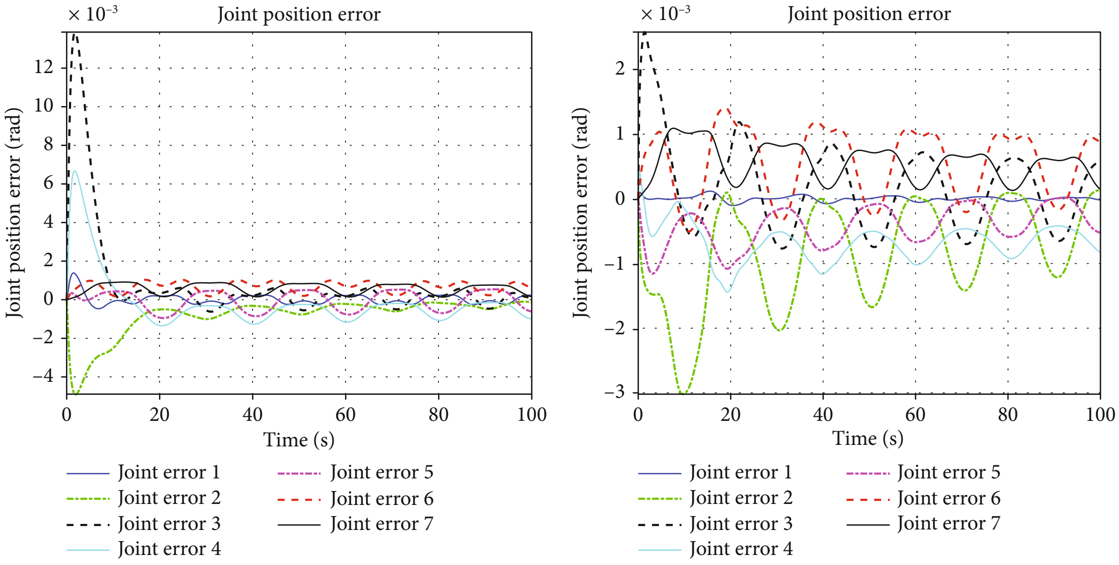

Figure 3: The tracking errors of the joint position: the direct adaptive controller (left) and the recursive composite adaptive controller (right).

where $\mathbf{P}_i \in \mathbb{R}^{13\times 13}$ is a positive definite matrix, $\mathbf{e}_j = \mathbf{W}_j\tilde{\theta}_j$, $\tilde{\theta}_j = \hat{\theta}_j - \theta_j$; defining $\mathbf{E} = \sum_{j=1}^{n}\mathbf{e}_j$ as the prediction error, Equation (23) is rewritten as follows:

$$\dot{\hat{\theta}}_i = -\mathbf{P}_i\left(\mathbf{Y}_{ri}{}^{T}\mathbf{s}_i + \gamma\mathbf{W}_i^{T}\mathbf{E}\right), \quad i = 1, 2, \cdots, n. \tag{24}$$

To sum up, Equations (18), (19a), (19b), (20), and (24) construct our recursive composite adaptive controller.

The stability verification of this recursive composite adaptive controller is also verified based on the subsystem dynamics. Considering the total Lyapunov function candidate as

$$\mathrm{V} = \sum_{i=0}^{n}\mathrm{V}_i, \tag{25}$$

where

$$\mathrm{V}_{\mathbf{i}} = \frac{1}{2}(\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}\mathbf{M}_i(\mathbf{V}_{ri} - \mathbf{V}_i) + \frac{1}{2}\tilde{\theta}_i^{T}\mathbf{P}_i^{-1}\tilde{\theta}_i. \tag{26}$$

Differentiating $\mathrm{V}_{\mathbf{i}}$ yields

$$\dot{\mathrm{V}}_{\mathbf{i}} = (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}\mathbf{M}_i\left(\dot{\mathbf{V}}_{ri} - \dot{\mathbf{V}}_i\right) + \tilde{\theta}_i^{T}\mathbf{P}_i^{-1}\dot{\tilde{\theta}}_i. \tag{27}$$

And according to the Appendix, it can be derived that

$$\begin{aligned}\dot{\mathrm{V}} &= -\sum_{i=1}^{n}(\mathbf{V}_{ri} - \mathbf{V}_i)^{T}\mathbf{K}_{Di}(\mathbf{V}_{ri} - \mathbf{V}_i) - \tilde{\theta}^{T}\mathbf{W}^{T}\mathbf{W}\tilde{\theta} \\ &= -\sum_{i=1}^{n}(\mathbf{V}_{ri} - \mathbf{V}_i)^{T}\mathbf{K}_{Di}(\mathbf{V}_{ri} - \mathbf{V}_i) - \mathbf{E}^{T}\mathbf{E} \le 0,\end{aligned} \tag{28}$$

with $\tilde{\theta} = [\tilde{\theta}_1^{T}\,\tilde{\theta}_2^{T}\,\cdots\,\tilde{\theta}_n^{T}]^{T}$, $\mathbf{W} = [\mathbf{W}_1 \quad \mathbf{W}_2 \quad \cdots \quad \mathbf{W}_n]$, and $\mathbf{E} = \mathbf{W}\tilde{\theta}$. $\dot{\mathrm{V}} \equiv 0 \longrightarrow (\mathbf{V}_{ri} - \mathbf{V}_i) \equiv 0$ for every $i = 1, 2, \cdots, n$ and $\mathbf{E} \equiv 0$, which means that the tracking error and the production error both globally asymptotically converge to 0.

Remark 1. The computational complexity of the proposed recursive composite adaptation is about $606n$ multiplications and $501n$ additions, which are much less than in [20] with $7730n^2 + 131n$ multiplications and $454.5n^2 - 82.5n$ additions. In the original composite controller [4], no consideration has been devoted to its computational aspects. Thus, computational complexity is no less than $O(n^4)$ since the computational complexity of the closed-form Lagrangian dynamics is $O(n^4)$. We have achieved the computational complexity of $O(n)$, which is at the same scale as the recursive direct adaptive controllers in [17–19].

4. Simulation Result

In this paper, the simulation results of our recursive composite adaptive control algorithm are presented in comparison with the direct adaptive control algorithm. The coordinate frames of the manipulator of the Chinese Space Station are plotted in Figure 2. The physical parameters are listed in Table 1. The load and the end-effector are connected in the final link, which is a combined link. The gravitational acceleration is assumed to be zero. The sampling period used in the simulation is 2 ms.

The parameters of the final row in Table 1 are set as $a_1 = 0.5$ m, $a_2 = 0.5$ m, $a_3 = 4$ m, $a_4 = 0.5$ m, $a_5 = 4$ m, $a_6 = 0.5$ m, $a_7 = 0.5$ m, and $a_8 = 1$ m.The desired velocity of each joint is set as $q_i = 5/180 * \pi * \sin(\pi/10t)$rad, $i = 1 \cdots n$. The initial joint position and velocity are set as $\mathbf{q}_0 = [0.3, \pi/2\,; \pi/3; -2\pi/3\,; \pi/3\,; \pi/2\,; 0.3]$rad and $\dot{\mathbf{q}}_0 = \mathbf{O}_{n\times 1}$, respectively. The controller parameter in Equations (19a), (19b) is set as $\mathbf{K}_{Di} = 10000 * \mathbf{I}_{6\times 6}$, $\Lambda_i = 1$. The initial value of the adaptation gain is set as $\mathbf{P}_{i,0} = 100 * \mathbf{I}_{13\times 13}$, $\gamma = 0.1$. The initial inertia

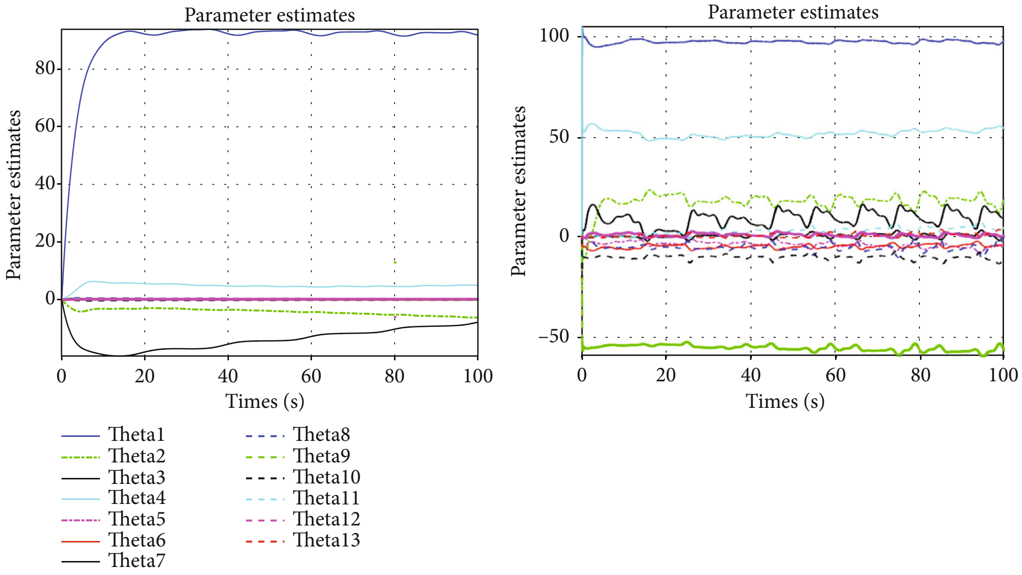

Figure 4: The parameter estimates of all the inertia parameters: the direct adaptive controller (left) and the recursive composite adaptive controller (right).

parameters are zeros. The tracking errors of the two methods are shown in Figure 3, and the parameter estimate figure is plotted in Figure 4. The results of all the links are similar; for briefness, only the parameter estimates of the final link have been shown in this paper.

Through the comparison of the direct adaptive controller, the tracking errors are obviously decreased by using the recursive composite adaptive controller. And the parameter estimates converge fast with the recursive composite adaptive controller.

5. Discussion

In this paper, a new recursive composite adaptive controller was proposed. The computational load is linear with the numbers of the joints, which is attractive especially for the redundant multijoint manipulator. The tracking errors are satisfied. And the convergence speed of the parameters of the proposed method is obvious. Additionally, the stability of the proposed algorithm is also proven based on the subsystem dynamics, which is more convenient.

Appendix

Proof of Stability

In Equation (27), $\mathbf{M}_i\dot{\mathbf{V}}_{ri}$ and $\mathbf{M}_i\dot{\mathbf{V}}_i$ can be replaced by the following equations, as follows:

$$\begin{aligned}\mathbf{M}_i\dot{\mathbf{V}}_{ri} &= \mathbf{F}_{ri}^* - \tilde{\mathbf{M}}_i \cdot \dot{\mathbf{V}}_{ri} - \left(\tilde{\mathbf{C}}_i + \mathbf{C}_i\right) \cdot \mathbf{V}_{ri} - \left(\tilde{\mathbf{G}}_i + \mathbf{G}_i\right) - \mathbf{K}_{Di}(\mathbf{V}_{ri} - \mathbf{V}_i),\\ \mathbf{M}_i \cdot \dot{\mathbf{V}}_i &= \mathbf{F}_i^* - \mathbf{C}_i \cdot \mathbf{V}_i - \mathbf{G}_i.\end{aligned} \tag{A1}$$

Equation (27) can be transformed as follows:

$$\begin{aligned}\dot{\mathrm{V}}_i = {}&-(\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}\mathbf{K}_{Di}\left(\dot{\mathbf{V}}_{ri} - \dot{\mathbf{V}}_i\right) + (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}(\mathbf{F}_{ri}^* - \mathbf{F}_r^*)\\ &- (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}\mathbf{Y}^T\widetilde{\theta} - (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}\mathbf{C}_i\left(\dot{\mathbf{V}}_{ri} - \dot{\mathbf{V}}_i\right) + \widetilde{\theta}_i^{\,T}\mathbf{P}_i^{-1}\dot{\widetilde{\theta}}_i,\end{aligned} \tag{A2}$$

where $(\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}\mathbf{C}_i(\dot{\mathbf{V}}_{ri} - \dot{\mathbf{V}}_i) = 0$,because $\mathbf{C}_i$ is an antisymmetric matrix. Substituting the adaptive control law Equation (24) into Equation (A2), the following equation is obtained:

$$\begin{aligned}\dot{\mathrm{V}}_i = {}&-(\mathbf{V}_{ri} - \mathbf{V}_i)^T\mathbf{K}_{Di}(\mathbf{V}_{ri} - \mathbf{V}_i) + (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}(\mathbf{F}_{ri}^* - \mathbf{F}_r^*)\\ &- \gamma\widetilde{\theta}_i^{\,T}\mathbf{W}_i^T\sum_{j=1}^{n}\mathbf{W}_j\widetilde{\theta}_j,\end{aligned} \tag{A3}$$

where

$$\begin{aligned}&(\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}(\mathbf{F}_{ri}^* - \mathbf{F}_r^*)\\ &\quad = (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}\left(\mathbf{F}_{ri} - {}^{i+1}T_i^T\mathbf{F}_{r,i+1} - \mathbf{F}_i + {}^{i+1}T_i^T\mathbf{F}_{i+1}\right)\\ &\quad = (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}(\mathbf{F}_{ri} - \mathbf{F}_i) - (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}\left({}^{i+1}T_i^T\mathbf{F}_{r,i+1} - {}^{i+1}T_i^T\mathbf{F}_{i+1}\right)\\ &\quad = (\mathbf{V}_{ri} - \mathbf{V}_i)^{\mathbf{T}}(\mathbf{F}_{ri} - \mathbf{F}_i) - (\mathbf{V}_{r,i+1} - \mathbf{V}_{i+1})^{\mathbf{T}}(\mathbf{F}_{r,i+1} - \mathbf{F}_{i+1})..\end{aligned} \tag{A4}$$

Therefore,

$$\begin{aligned}\dot{V} = &-\sum_{i=1}^{n} (\mathbf{V}_{ri} - \mathbf{V}_i)^T \mathbf{K}_{Di} (\mathbf{V}_{ri} - \mathbf{V}_i) + (\mathbf{V}_{r0} - \mathbf{V}_0)^{\mathbf{T}} (\mathbf{F}_{r0} - \mathbf{F}_0) \\ &- (\mathbf{V}_{r,n+1} - \mathbf{V}_{n+1})^{\mathbf{T}} (\mathbf{F}_{r,n+1} - \mathbf{F}_{n+1}) \\ &- \gamma \begin{bmatrix} \widetilde{\theta}_1^T & \widetilde{\theta}_2^T & \cdots & \widetilde{\theta}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{W}_1^T \\ \mathbf{W}_2^T \\ \vdots \\ \mathbf{W}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 & \cdots & \mathbf{W}_n \end{bmatrix} \begin{bmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \\ \vdots \\ \widetilde{\theta}_n \end{bmatrix} \\ = &-\sum_{i=1}^{n} (\mathbf{V}_{ri} - \mathbf{V}_i)^T \mathbf{K}_{Di} (\mathbf{V}_{ri} - \mathbf{V}_i) - \widetilde{\theta}^T \mathbf{W}^T \mathbf{W} \widetilde{\theta} \le 0, \end{aligned} \tag{A5}$$

where $(\mathbf{V}_{r0} - \mathbf{V}_0) = 0$ because the velocity of the base of the manipulator is zero. $(\mathbf{F}_{r,n+1} - \mathbf{F}_{n+1}) = 0$, because there is no external force acting on the final link of the manipulator.

Data Availability

The simulation date and the program files in this paper cannot be shared with others as they are our basis for next research.

Conflicts of Interest

The authors declared that they have no conflicts of interest to this work.

Acknowledgments

This work was supported by the Major Project of the New Generation of Artificial Intelligence (No. 2018AAA0102900).

References

[1] Y. Lin, D. Li, Y. Wang, and Y. Wang, *Current Status and Analysis of Space Robot*, Spacecraft Engineering, 2015.

[2] L. Daming, R. Wei, H. Chengwei, W. Yaobin, T. Zixin, and W. Youyu, "Key Technology Review of the Research on the Space Station Manipulator," *Manned Spaceflight*, vol. 20, no. 3, pp. 238–242, 2014.

[3] J. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, Englewood Cliffs, NJ, 1991.

[4] J. Slotine and W. Li, "Composite adaptive control of robot manipulators," *Automatica*, vol. 25, no. 4, pp. 509–519, 1989.

[5] A. Belherazem and M. Chenafa, "Passivity based adaptive control of a single-link flexible manipulator," *Automatic Control and Computer Sciences*, vol. 55, no. 1, pp. 1–14, 2021.

[6] X. Li, Y. Pan, G. Chen, and H. Yu, "Adaptive human–robot interaction control for robots driven by series elastic actuators," *IEEE Transactions on Robotics*, vol. 33, no. 1, pp. 169–182, 2017.

[7] Y. Li, C. Yang, W. Yan, R. Cui, and A. Annamalai, "Admittance-based adaptive cooperative control for multiple manipulators with output constraints," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 12, pp. 3621–3632, 2019.

[8] F. Wang, Z. Liu, C. Chen, and Y. Zhang, "Synchronized adaptive control for coordinating manipulators with time-varying actuator constraint and uncertain dynamics," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 12, pp. 4149–4167, 2019.

[9] L. Yan, T. Stouraitis, and S. Vijayakumar, "Decentralized ability-aware adaptive control for multi-robot collaborative manipulation," *IEEE Robotics and Automation Letters*, vol. 6, no. 2, pp. 2311–2318, 2021.

[10] Y. Ren, S. Sosnowski, and S. Hirche, "Fully distributed cooperation for networked uncertain mobile manipulators," *IEEE Transactions on Robotics*, vol. 36, no. 4, pp. 984–1003, 2020.

[11] V. Yen, W. Nan, and P. van Cuong, "Recurrent fuzzy wavelet neural networks based on robust adaptive sliding mode control for industrial robot manipulators," *Neural Computing and Applications*, vol. 31, no. 11, pp. 6945–6958, 2019.

[12] A. Izadbakhsh, S. Khorashadizadeh, and S. Ghandali, "Robust adaptive impedance control of robot manipulators using Szasz-Mirakyan operator as universal approximator," *ISA Transactions*, vol. 106, pp. 1–11, 2020.

[13] H. Huang, C. Yang, and C. Chen, "Optimal robot-environment interaction under broad fuzzy neural adaptive control," *IEEE Transactions on Cybernetics*, vol. 51, no. 7, pp. 3824–3835, 2021.

[14] D. Wang and S. Zhang, "Improved neural network-based adaptive tracking control for manipulators with uncertain dynamics," *International Journal of Advanced Robotic Systems*, vol. 17, no. 4, article 172988142094756, 2020.

[15] J. Slotine and Weiping Li, "On the adaptive control of robot manipulators," *International Journal of Robotics Research*, vol. 6, no. 3, pp. 49–59, 1987.

[16] R. Featherstone and E. Univ, *Robot Dynamics Algorithms*, Kluwer, 1987.

[17] G. Niemeyer and J. Slotine, "Performance in adaptive manipulator control," in *Proceedings of the 27th IEEE Conference on Decision and Control*, pp. 1585–1591, Austin, TX, USA, 1991.

[18] W. Huo and W. Gao, "A new robot model and associated control algorithm," *Acta Automatica Sinica*, vol. 20, no. 3, pp. 278–285, 1994.

[19] W. Zhu, *Virtual Decomposition Control*, Springer, Berlin Heidelberg, 2010.

[20] H. Wang, "Recursive composite adaptation for robot manipulators," *Journal of Dynamic Systems, Measurement, and Control*, vol. 135, no. 2, article 021010, 2013.

[21] M. Spong and R. Ortega, "On adaptive inverse dynamics control of rigid robots," *IEEE Transactions on Automatic Control*, vol. 35, no. 1, pp. 92–95, 1990.